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Fabio Freitas, Esther Dweck y Fernando Ligiéro

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Tel.: (+5411) 4348-3582/3814
Email: ensayos.economicos@bcra.gob.ar
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Autonomous Demand Composition and Fiscal Policy in a Supermultiplier Simulation Model

Fabio Freitas
Esther Dweck
Fernando Ligiéro*
Federal University of Rio de Janeiro, Brazil

Abstract

The paper aims to analyze the influence of autonomous demand composition on the impact of fiscal policy on the economic performance of an economy. To do so, we develop a supermultiplier simulation model for a small open economy. In our simulations, we use two sets of exogenous variables and parameter combinations. The main differences between these two sets are that in Country 1, the government expenditure share in total autonomous demand is relatively high, and the economy is more inward-oriented. In contrast, in Country 2, the government expenditure share is relatively low, and the economy is more outward-oriented. We then simulate the impact of the same temporary shock on GDP growth and the debt to GDP ratio in both countries of a fiscal policy that follows a stylized structural balance rule. The main result obtained from the simulation exercise points to the importance of the government share in total autonomous expenditure as the factor explaining the diverging economic performance regarding GDP growth and the debt to GDP ratio under the same fiscal policy.

JEL Classification: E12, E13, E62, H63.

Keywords: debt, fiscal policy, modeling, small open economy, supermultiplier.

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Composición de la demanda autónoma y política fiscal en un modelo de simulación con supermultiplicador

Fabio Freitas
Esther Dweck
Fernando Ligiéro
Universidad Federal de Rio de Janeiro, Brasil

Resumen

Este documento tiene como objetivo analizar la influencia de la composición de la demanda autónoma sobre el impacto de la política fiscal en el desempeño de una economía. Para ello, desarrollamos un modelo de simulación de supermultiplicador para una economía pequeña y abierta. En nuestras simulaciones, utilizamos dos conjuntos de variables exógenas y combinaciones de parámetros. Las principales diferencias entre estos dos conjuntos son que en el País 1, la participación del gasto público en la demanda autónoma total es relativamente alta y la economía está más orientada hacia adentro. En contraste, en el País 2, la participación del gasto público es relativamente baja y la economía está más orientada hacia el exterior. Luego, simulamos el impacto del mismo *shock* temporario de una política fiscal que sigue una regla estándar de balance estructural sobre el crecimiento del PIB y el ratio de deuda a PIB en ambos países. El principal resultado obtenido del ejercicio de simulación apunta a la importancia de la participación del gobierno en el gasto total autónomo como factor que explica el desempeño económico divergente en cuanto al crecimiento del PIB y a la relación deuda/PIB bajo la misma política fiscal.

Clasificación JEL: E12, E13, E62, H63.

Palabras clave: deuda, economía pequeña y abierta, modelación, política fiscal, supermultiplicador.

1. Introduction

The paper aims to analyze the influence of autonomous demand composition on the impact of fiscal policy on the economic performance of an economy. To do so, we will present a supermultiplier simulation model for a small open economy. We will undertake a pure demand-led growth simulation exercise that ignores, for analytical purposes only, the possible constraints to a demand-led growth pattern associated with the balance of payments, labor force employment dynamics, natural resources, and environmental issues. The simulation exercise is based on two sets of exogenous variables and parameter combinations. The main differences between these two sets are that in Country 1, the government expenditure share in total autonomous demand is relatively high, and the economy is more inward-oriented. In contrast, in Country 2, the government expenditure share is relatively low, and the economy is more outward-oriented. In our simulation exercise, we suppose the existence of an underlying economic scenario capturing a situation in which the economy is subject to a "temporary shock" involving changes in the growth rate of private autonomous expenditure, the interest rate, and functional distribution of income. We also suppose that the governments in the two countries follow the same stylized fiscal policy rule, a structural balance rule. The paper's main conclusion is that, within our analytical framework, the composition of autonomous demand has important effects on the economy's performance and, in particular, on the level and rate of growth of GDP and the debt to GDP ratio.

The structure of the paper is the following. The first section presents the supermultiplier growth model that will be used in our simulation exercise. The second section discusses the calibration of the parameters/exogenous variables of the model and the initial conditions for the endogenous variables. Next, the economic scenario and stylized fiscal policy rule used in the simulation are presented in the paper's third section. The fourth section presents and discusses the main results obtained from the simulation exercise. Finally, we present our concluding remarks highlighting the main result obtained and indicating how its robustness can be evaluated within our analytical framework.

2. A supermultiplier simulation model for a small open economy with a government sector

We develop a simple version of the supermultiplier model for the simulation analysis of the impact of fiscal policy on a small economy.[1] As a first approximation, we will explore only the pure demand-led pattern of economic growth in our simulations, leaving entirely out of the picture the main constraints to demand-led growth: the balance of payments, the labor force, and natural resources. To further simplify the model, we suppose that government activities are limited to government consumption and investment, social transfers, direct taxation, and the issuing of domestic denominated debt. The monetary authority controls the real rate of interest through the

[1] The specific version here developed has Freitas and Christianes (2020) and Haluska *et al*. (2019) as its main references in the literature.

manipulation of the nominal interest rate. Income distribution is exogenously determined, reflecting the influence of class conflict mediated by the institutional and social framework. Moreover, we also assume that the only production method in use requires a fixed combination of a homogeneous labor input with homogeneous fixed capital to produce a single product. Constant returns to scale prevail, and there is no technological progress. Finally, the model is specified in real terms, in the sense that all relevant variables of the model are real magnitudes and in discrete time.[2]

Let us start by supposing that an equilibrium between aggregate supply and demand, which in our open economy with a government setting, implies that:

$$Y_t + M_t = C_t + I_t + G_t + X_t \quad (1)$$

where Y is the gross domestic product (GDP), M imports, C household consumption, I aggregate investment, G government consumption, and X exports.

We suppose that total imports are a function of aggregate demand and, for simplicity, we also assume a uniform import content coefficient for all aggregate demand components. Therefore, we have:

$$M_t = m(C_t + I_t + G_t + X_t) \quad (2)$$

where m is the import content coefficient, with $0 \leq m \leq 1$.[3]

Household consumption depends on three sources: the purchasing power introduced in the economy by capitalist production decision to mobilize the labor force, as captured by the wage bill; the purchasing power introduced in the economy through government social transfers; and the purchasing power introduced in the economy by the consumption expenditures financed by credit or by previously accumulated wealth. Consumption financed by wages is an induced expenditure since it depends on current production decisions. As a first approximation, we suppose that the marginal propensity to consume out of wages is equal to one and, therefore, consumption out of wages is equal to the after-tax wage bill. Thus, given the wage share on total income (denoted ω below) and tax rate on wages (denoted t_w below), consumption out wages is proportional to GDP. On the other hand, consumption out of social transfers is an autonomous source of demand in the economy, and we assume, as a first approximation, that all social transfers are spent. Finally, consumption financed by credit and accumulated wealth (denoted C^A below) is also an autonomous source of demand.[4] Therefore, aggregate consumption is given by:

[2] Throughout the article we denote x_t as the value of the variable x at the finite time period t.

[3] See Miyazawa (1976, chap. 3) and Gandolfo (2002, chap. 8, sec. 8.4 and appendix D, sec. D.3) for this kind of formulation of the import function. Notice that the import content coefficient has limits to its variations, which excludes the possibility of continual increasing or decreasing. Note also that it includes the derived demand for imported intermediate inputs.

[4] As a first approximation, rentier and capitalist consumption are completely autonomous in this formulation, that is, the marginal propensity to consume out of profits and government debt repayments (interest and principal) is zero.

$$C_t = (1 - t_w)\omega Y_t + Tr_t + C_t^A \tag{3}$$

where Tr_t is the value of social transfers as for period t.

On the other hand, total investment has four components, of which three of them are supposed to be autonomous: public investment, I^G; residential investment by households, I^R; R&D expenditures by capitalist firms, I^{AF}. The last component is the induced investment by capitalist firms, I_t^{IF}. We assume that the capital stock adjustment principle explains the latter. According to this principle, capitalist competition influences the investment process by bringing about a tendency towards the adjustment of productive capacity to the production flows required to meet demand at a price that covers production expenses and allows, at least, the obtainment of a minimum required profitability. Since capacity adjustment is not instantaneous due to technical and economic indivisibilities and firms do not want to lose their market shares to incumbent firms and potential entrants, they maintain margins of spare capacity to allow the adjustment of production to a fluctuating demand. On the other hand, under the pressure of competition, firms also do not want to keep accumulating costly unneeded spare capacity when the actual degree of capacity utilization remains below the profitable normal level. In this case, a reduction in the pace of investment in relation to demand increases capacity utilization and the realized rate of profit without putting in danger the firm's market share. Thus, we arrive at the notion of a planned margin of spare capacity that corresponds to a normal capacity utilization rate. Following Ciccone (1986; 1987), we interpret the normal or planned rate of capacity utilization as determined, among other things, by the historically 'normal' ratio of peak to average demand. This latter ratio is assumed to be unaffected by current oscillations of demand since it is presumably based on the observation of the actual cyclical and seasonal patterns of the market over a long period. To represent this kind of investment behavior, we use the following investment function close to the specification suggested by Serrano (1995a; 1995b) and Cesaratto *et al*. (2003):

$$I_t^{IF} = v\,(\delta + g_t^e)Y_t \tag{4}$$

where v is the normal capital-output ratio, δ is the capital replacement coefficient, and g_t^e is the expected GDP growth rate at period t. We further assume that expectations are formed according to an adaptive pattern, so that:

$$g_t^e = (1 - x)g_{t-1}^e + x g_{t-1} \tag{5}$$

where x is the expectation adjustment parameter, with $0 \leq x \leq 1$.

From the assumptions above, we obtain the equilibrium level of GDP of the economy as follows:

$$Y_t = \left\{\frac{1 - m}{1 - (1 - m)[(1 - t_w)\omega + v\,(\delta + (1 - x)g_{t-1}^e + x g_{t-1})]}\right\} Z_t \tag{6}$$

where the term in braces is the supermultiplier incorporating both consumption and investment inducement effects and $Z_t = G_t + I_t^G + Tr_t + I_t^R + I_t^{AF} + X_t + C_t^A$ is the autonomous component of aggregate demand. The rate of growth of autonomous demand is the weighted average of the growth rates of its components, so that:

$$g_{Zt} = \left(\frac{G_{t-1}}{Z_{t-1}}\right) g_{Gt} + \left(\frac{I_{t-1}^G}{Z_{t-1}}\right) g_{I^G t} + \left(\frac{Tr_{t-1}}{Z_{t-1}}\right) g_{Trt} + \left(\frac{I_{t-1}^R}{Z_{t-1}}\right) g_{I^R t} + \left(\frac{I_{t-1}^{AF}}{Z_{t-1}}\right) g_{I^{AF} t} + \left(\frac{C_{t-1}^A}{Z_{t-1}}\right) g_{C^A t} + \left(\frac{X_{t-1}}{Z_{t-1}}\right) g_{Xt} \tag{7}$$

The growth rates of the various autonomous demand components will be discussed below in connection with the definition of the fiscal policy rule and economic scenarios for the simulation exercises. From (6) and (7), we obtain a difference equation for the rate of growth of GDP according to the model:

$$g_t = g_{Zt} + \frac{(1-m)v\,(1+g_{Zt})x(g_{t-1} - g_{t-1}^e)}{1-(1-m)[(1-t_w)\omega + v\,(\delta + (1-x)g_{t-1}^e + xg_{t-1})]} \tag{8}$$

The GDP growth rate depends on the rate of growth of autonomous demand and on the change in the supermultiplier caused by the revisions of growth expectations that affect the propensity to invest (i.e., $v\,(\delta + g_t^e)$).[5]

Additionally, when exogenous changes of some variables present in the supermultiplier occur, there is a temporary change in GDP growth rate and, therefore, a level effect on GDP. In our simulations, we will restrict our analysis to changes of the wage share on total income (denoted ω_t below) and of the income tax rate on wage income (t_{wt}). These changes are captured on the RHS of equation (8). For each specific source of change, we have an additional term like the second one on the RHS of equation (8).

Regarding the fiscal policy indicators, we will discuss the behavior of the primary surplus to GDP ratio and debt to GDP ratio. The primary surplus to GDP ratio is defined as follows:

$$\frac{\beta_t}{Y_t} = \frac{T_t - G_t - I_t^G - Tr_t}{Y_t} = \frac{t_w \omega Y_t + t_K(1-\omega)Y_t + t_r r B_{t-1} - G_t - I_t^G - Tr_t}{Y_t} \tag{9}$$

[5] Equations (5) and (8) form together a 2x2 system of difference equations in g and g^e. Under the assumption that all autonomous demand components grow at the same constant rate g_Z the system has a steady-state equilibrium in which $g^* = g^{e*} = g_Z$ and this equilibrium is locally dynamically stable if the disequilibrium propensity to spend has a value lower than one in the neighborhood of the equilibrium, that is if $(1-m)[(1-t_w)\omega + v(g_{zt}+\delta) + vx + vxg_{zt}) < 1$. See Serrano *et al*. (2019, Appendix B) for a formal demonstration of these results in the case of a closed economy without government.

where β is the primary surplus, t_K is the tax rate on profits, t_r is the tax rate on interest received by public debt owners, r is the real interest rate, B is the public debt stock. Moreover, we assume, for simplicity, that $t_w = t_K = t_r$.

On the other hand, the following difference equation gives us the dynamic behavior of the public debt to GDP ratio (denoted b below):

$$b_t = -\frac{\beta_t}{Y_t} + \left(\frac{1+r}{1+g_t}\right) b_{t-1}$$

or

$$b_t = \frac{G_{t-1}(1+g_{Gt}) + I_{t-1}^G\left(1+g_{I^G t}\right) + Tr_{t-1}(1+g_{Trt})}{Y_{t-1}(1+g_t)} - t_w\omega - t_K(1-\omega) + \left(\frac{1+(1-t_r)r}{1+g_t}\right) b_{t-1} \quad (10)$$

Note that if a transformed Domar condition $g_t > (1-t_r)r$ holds, it is possible to maintain a primary deficit to GDP ratio (excluding interest tax revenues) without an increasing or decreasing debt to GDP ratio. Conversely, if $g_t < (1-t_r)r$, a stationary debt to GDP ratio requires the maintenance of a sufficiently high primary surplus to GDP ratio.[6]

3. Model calibration and initial values

The calibration of the parameters/exogenous variables of the model and the definition of the initial values for endogenous variables are chosen to reflect two different kinds of open economies. In Country 1, the government expenditure share in total autonomous demand is relatively high, and the economy is more inward-oriented. In contrast, in Country 2, the government expenditure share is relatively low, and the economy is more outward-oriented.

In the case of Country 1, the model calibration and initial values were set to reflect, whenever possible, the Brazilian economy.[7] On the other hand, in the case of Country 2, the calibration and initial values were set to the same values as Country 1, whenever possible, but changed to reflect a much smaller government sector and a lot larger foreign trade relative to the size of the economy. In both countries, the initial level of GDP is set at 100 to facilitate comparison between both countries and, hence, initial levels of autonomous expenditure represent percentages of GDP. Despite the differences between both economies, parameters/exogenous variables and

[6] In particular, assuming a given primary surplus to GDP ratio (excluding the tax revenues on interest) β' and that all autonomous demand components grow at the same given rate g_Z, the steady-state equilibrium for equation (10) above is $b^* = -(\beta'/Y)^*/(g_Z - (1-t_r)r)/(1+g_Z)$. Under these assumptions this equilibrium is stable if the (Domar) condition $g_Z > (1-t_r)r$ holds.

[7] The parameters/exogenous variables and initial values for endogenous variables were based on data from the Complete Tables of the Quarterly National Accounts System up to 2019 and yearly data of the Complete Tables of the National Accounts System up to 2018 (IBGE 2021a, IBGE 2021b).

initial values of the endogenous variables (presented in Table 1 and Table 2 below) are chosen so that both countries have precisely the same initial level of autonomous expenditure in order to focus the analysis on the role of the autonomous demand composition.

Table 1 | Calibration of parameters/exogenous variables for the simulation exercise

Parameter	Value		Description
	Country 1	Country 2	
m	0.15	0.31	Propensity to import.
x	0.10		Sensitivity of fixed private investment to the difference between current and expected demand.
ω	0.57		Pre-tax labor share in income.
v	1.65		"Normal" capital to output ratio.
δ	6.6%		Annual replacement coefficient.

Source: Own calculation based on IBGE (2021a, 2021b), Souza and Cornelio (2020), and Haluska et al. (2019)

Although the calibration and initial variables for Country 1 were set in reference to real Brazilian data whenever possible, given the high degree of abstraction of the model regarding taxation, the rate of direct taxation is set in a somewhat arbitrary fashion, being much higher than occurs in reality. Also, since some of the more interesting results of the model are apparent only when the economy is close to fulfilling the transformed Domar condition for a balanced debt to GDP ratio, taxation was set so that there is a primary surplus of 1% of GDP at $t = 1$. Since the real interest rate begins at 4% a year, that means, for example, that a yearly growth rate of 3% of GDP leads to a falling ratio of debt to GDP.[8]

Initial real interest rates of 4% per year are close to rates for 10-year government debt currently in Brazil, which according to ANBIMA (2021), stood at 3.9603% for IPCA (the official consumer price index) inflation-indexed bonds on April 29th, 2021. The real rate of interest applicable to gross debt is considerably lower, mainly because government bonds used in monetary policy repo transactions by the Central Bank constitute a large share of this debt, with nominal rates for gross debt at 5.8% a year for an annual inflation rate of 6.10% in March 2021 (BCB, 2021; IBGE, 2021c). Given these numbers and the history of Brazilian real interest rates, the simulation involves real interest rates between 4% and 6 %, as will be commented below, which seem plausible for the Brazilian Economy.

[8] The exact rate above which GDP must grow in order to lower the debt to GDP ratio also depends on the initial debt to GDP ratio as can be seen in equation (10).

Table 2 | Initial values of endogenous variables for the simulation exercise

Variable	Initial Value		Description
	Country 1	Country 2	
$t_{wt=1}$	0.40	0.22	Rate of direct taxation on labor income in *t* = 1.
$t_{Kt=1}$	0.40	0.22	Rate of direct taxation on capital income in *t* = 1.
$t_{rt=1}$	0.40	0.22	Rate of direct taxation on rentier income in *t* = 1.
$r_{t=1}$	4%		Real interest rate in *t* = 1.
$b_{t=1}$	90%		Level of the debt to GDP ratio in *t* = 1
$\frac{\beta}{Y}$	1%		Primary surplus β as a % of GDP Y in *t* = 1.
$g_{t=1}$	2.0%		GDP growth rate in *t* = 1.
$g^e_{t=1}$	2.5%		Expected GDP growth rate in *t* = 1.
$g^Z_{t=1}$	2.0%		Autonomous expenditure growth rate in *t* = 1.
$Y_{t=1}$	100		Level of GDP normalized to 100 in *t* = 1.
$X_{t=1}$	14	34	Level of exports of goods and services in *t* = 1.
$G_{t=1}$	20	10	Level of government transfers in *t* = 1.
$Tr_{t=1}$	18.5	9.5	Level of government transfers excluding interest in *t* = 1.
$I^G_{t=1}$	2	1	Level of government investment in *t* = 1.
$ZG_{t=1}$	40.5	20.5	Level of government autonomous expenditure in *t* = 1.
$I^R_{t=1}$	4.5	4.5	Level of residential investment in *t* = 1.
$I^{AF}_{t=1}$	3	3	Level of autonomous firm investment in *t* = 1.
$C^A_{t=1}$	6.4	6.4	Level of autonomous consumption in *t* = 1.
$Z_{t=1}$	68.4	68.4	Level of autonomous expenditure in *t* = 1.
$ZG_{t=1}/Z_{t=1}$	59.2%	30.0%	Share of government autonomous expenditure in total autonomous expenditure in *t* = 1.

Source: own calculation based on IBGE (2021a, 2021b) and BCB (2021).

Regarding the parameter that reflects the adjustment of expected demand to past growth that is used to determine fixed-capital private investment, we suppose that $x = 0.10$, which Haluska *et al*. (2019, p. 30, note 16) point as a possible benchmark in the existing literature. This value is well above $x = 0.07$, the value obtained in the econometric estimation in the same paper (Ibid., pp. 27-32) for the United States from 1985 to 2017. These values for the parameter x are sufficiently low to avoid the likelihood of unstable behavior of the model, even for relatively high growth rates of the economy.

For the exogenous variable v, the desired or "normal" capital-output ratio, we use the estimates for the capital stock of the Brazilian economy, excluding the stock of residential capital obtained by Souza and Cornelio (2020, pp. 44-45). Thus, considering the average value of v between 1998-2017 as a proxy, we use $v = 1.65$ in our simulations. The latter value is higher than the one ($v = 1.07$) obtained by Haluska *et al*. (2019, p. 30) for the U. S. economy. However, it is important to note that even if our estimate is somewhat inaccurate, reasonable calibrations of this parameter do not significantly change the simulation results.

As a proxy for δ, which is the capital replacement coefficient, we utilize the depreciation rate of fixed capital. Once more, we use the estimates of Souza e Cornelio (2020, pp. 44-45) for the Brazilian Economy, in which the average rate of depreciation has grown in recent years because of the subitem "other" capital stock, that includes intellectual property products, which constitute a small but growing share of the capital stock.[9] With this observation in mind, we considered only the average depreciation from 2013 to 2017, obtaining $\delta = 6.6\%$ as a proxy. This result is very close to the estimate obtained for the U. S. economy by Haluska *et al*. (2019. p. 30). Just as in the calibration of v, even if our estimate is somewhat inaccurate, reasonable calibrations of this parameter do not significantly change the simulation results.

Given the parameters and initial conditions regarding the primary surplus and real interest rate, the crucial initial condition is the one capturing the composition of autonomous demand. Both countries have the same initial level of total autonomous expenditure. Still, as illustrated in the last line Table 2, this is set so that Country 1 has 59.2% of autonomous expenditure being government expenditure, and in Country 2 the same variable is 30.0% percent. This is the crucial supposition that will explain the different results obtained in the simulations for both countries, which otherwise share very similar conditions.

4. Economic simulation scenario and stylized fiscal policy rule

We illustrate some features of the model by simulating a scenario with a "temporary shock" involving a change to the growth rate of private autonomous expenditure, interest rates, and the functional distribution of income. These variables go through marked changes that last for a few years and then return to their previous magnitudes. A structural balance fiscal policy rule has been applied to this scenario, seeking to lower the debt to GDP ratio starting in year $t = 2$.

The scenario was set to reflect the example of a country in the periphery suffering a temporary shock that lasts for a few years, which initially affects exports and then leads to higher interest rates, negative growth rates for other autonomous private expenditures, and a change in the functional distribution that favors capital over labor. The exogenously determined growth rates of exports and private autonomous demand components, the interest rate, and the government share of total autonomous demand in the scenario are presented in Appendix 1.

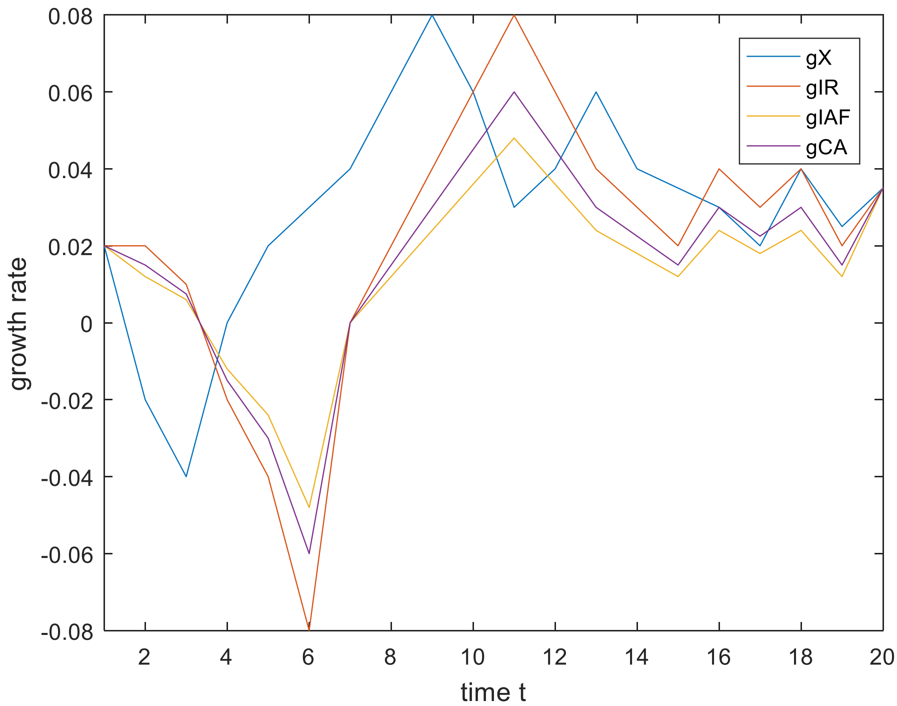
The shock to exports, which were growing at $gX = 2\%$ at $t = 1$, leads to negative growth rates in $t = 2$ and $t = 3$ and exports staying flat in $t = 4$ with zero growth. Exports then recover so that exports grow at an approximately 3% per year geometric average rate for the entire simulation period of twenty years. Real interest rates increase, starting in $t = 3$, until they reach 6% in $t = 6$, then fall back starting in $t = 7$ until they reach 4% a year in $t = 10$ and remain at this level until the end of the simulation period of twenty years. The functional distribution of income gradually becomes more favorable to capitalists in relation to workers starting in $t = 3$ with $\omega = 57\,\%$ in $t = 1$ changing to $\omega = 55\%$ in $t = 6$, then gradually increasing back, beginning in $t = 7$, until it

[9] In our calculations of the depreciation rate, we did not take into account the stock of residential capital.

stabilizes when reaching $\omega = 57\,\%$ in $t = 10$ and after that remaining in this level until the end of the simulation period. Other private autonomous expenditure growth rates (excluding exports) are set exogenously so that they are inversely related to real interest rates, but with residential investment being more sensitive than autonomous consumption and this last expenditure being more sensitive than autonomous firm investment.

Graph 1 below illustrates the behavior of the growth rate of autonomous expenditure for the entire period, with gX, gIR, $gIAF$, and gCA corresponding to the growth of exports, residential investment, autonomous firm expenditure, and autonomous consumption, respectively.

Graph 1 | Growth rates of private autonomous demand components



Source: own calculations.

Regarding the stylized fiscal policy rule, the structural balance rule requires, initially, a primary structural surplus to GDP target ratio to set the rule. This target is set at 2.5% of potential GDP. Since the initial primary balance is 1% and is supposed to be equal to the initial primary balance at potential GDP, there must be a fiscal adjustment involving 1.5% of GDP. The structural balance rule does not prescribe how this adjustment must be conducted, something that in practice would be up to policymakers to decide. We suppose this adjustment occurs with 2/3 of the adjustment falling on projected spending and 1/3 on raising revenue. Table 3 below summarizes this procedure.

Table 3 | Structural primary surplus target and fiscal adjustment

% of GDP				
Primary Balance at *t*=1	Structural Primary Surplus Target ($\frac{\beta}{Y_t^e}$)	Total fiscal Adjustment	Adjustment Apportionment	
			Δ spending (2/3)	Δ revenue (1/3)
1.0%	2.5%	1.5%	1.0%	0.5%

Now, it is necessary to proceed with the adjustment by altering projected spending and revenue (supposing that spending growth was the same as expected GDP growth and considering the revenue expected at this growth rate). We have that $\Delta\, spending$ is expressed as % of GDP and $\Delta\, spending^{adj}$ is merely a variable used to transform spending from % of GDP to a factor that adjusts government spending growth:

$$original\ projected\ spending = ZG_t^e = (I_t^G + G_t + Tr_t)\,(1 + g_t^e) \Longrightarrow$$

$$new\ spending\ growth = (I_t^G + G_t + Tr_t)\,(1 + g_t^e - \Delta\, spending_t^{adj}) \Longrightarrow$$

$$\Delta\, spending^{adj} = \left[\frac{(Y_{t-1})(1 + g_t^e)}{G_{t-1} + IG_{t-1} + Tr_{t-1}}\right] \Delta\, spending \qquad (11)$$

Equation (11) shows that the adjustment in the rate of spending growth is the same as the projected spending cut as a percentage of projected GDP multiplied by the term in brackets.[10] Regarding the necessary adjustment in projected revenue, we have:

$$original\ projected\ revenue = T_t^e = (Y_{t-1})(1 + g_t^e)[t_{wt}\omega + t_{Kt}(1-\omega)] + t_r r B_{t-1}$$

However, since we supposed that $t_{wt} = t_{Kt} = t_{rt}$, then:

$$original\ projected\ revenue = T_t^e = (Y_{t-1})(1 + g_t^e)[t_{wt}\omega + t_{wt}(1-\omega)] + t_{wt} r B_{t-1}$$

$$\Longrightarrow T_t^e = \{(Y_{t-1})(1 + g_t^e)[\omega + (1-\omega)] + rB_{t-1}\}t_{wt} \Longrightarrow$$

[10] Since $\Delta\, \mathrm{spending}^{\mathrm{adj}} \left[\frac{G_{t-1}+IG_{t-1}+Tr_{t-1}}{(Y_{t-1})(1+g_t^e)}\right] = \Delta\, \mathrm{spending}$. In our example, we have $\Delta\, \mathrm{spending} = 1\%\ \mathrm{of\ projected\ GDP} = 1\% * (Y_{t-1})(1 + g_t^e)$, that needs to be transformed into a deeper cut in spending than 1%, since government spending is only a fraction of GDP. Since the term in brackets is smaller than 1, that is $\left[\frac{G_{t-1}+IG_{t-1}+Tr_{t-1}}{(Y_{t-1})(1+g_t^e)}\right] < 1$, then when $\Delta\, \mathrm{spending}$ is divided by the term in brackets we have $\Delta\, \mathrm{spending}^{\mathrm{adj}}$, which is necessarily larger than 1%, that is, a 1% adjustment of GDP in spending implies a much larger adjustment in the growth percentage of spending.

$$T_t^e = \{(Y_{t-1})(1 + g_t^e) + rB_{t-1}\}t_{wt}$$

$$original\ projected\ revenue\ \%\ of\ GDP = t_t^e \Longrightarrow$$

$$t_t^e = [1 + \frac{rb_{t-1}}{1 + g_t^e}]t_{wt} \Longrightarrow$$

$$original\ projected\ revenue +\ new\ required\ revenue\ \%\ of\ GDP =$$

$$t_t^e + \Delta\ \text{revenue} =$$

$$[1 + \frac{rb_{t-1}}{1 + g_t^e}]t_{wt}^{adjustment}$$

$$\Longrightarrow t_{wt}^{adjustment} = \frac{t_t^e + \Delta\ \text{revenue}}{1 + \frac{rb_{t-1}}{1 + g_t^e}} \Longrightarrow$$

$$t_{wt}^{adjustment} = t_{wt} + \frac{\Delta\ revenue}{1 + \frac{rb_{t-1}}{1 + g_t^e}} \tag{12}$$

Equation (12) shows that the adjustment in the income tax rate applicable to workers income, which out of simplicity we supposed was equal to the income tax rate applicable to capitalists and rentiers ($t_w = t_K = t_r$), is the original worker tax rate, adjusted by the increase in projected revenue required by the fiscal rule as a percentage of GDP, divided by the term $1 + \frac{rb_{t-1}}{1+g_t^e} > 1$. This adjustment occurs, so a required increase in revenue as a percentage of GDP ($\Delta\ revenue$) is expressed as an adjustment in the tax rate $t_{wt} = t_{Kt} = t_{rt}$.[11]

The above formulation could be used for different types of fiscal rules involving different references regarding GDP growth. But for the structural balance rule here used, this should be the expected growth rate of potential GDP. To do this, a simple rule is adopted that uses the same rate of expected GDP growth as the private fixed investment adaptive expectation rule:

$$gstruct_t^e = g_t^e, \tag{13}$$

with:

$$g_t^e = (1 - x)g_{t-1}^e + xg_{t-1} \tag{14}$$

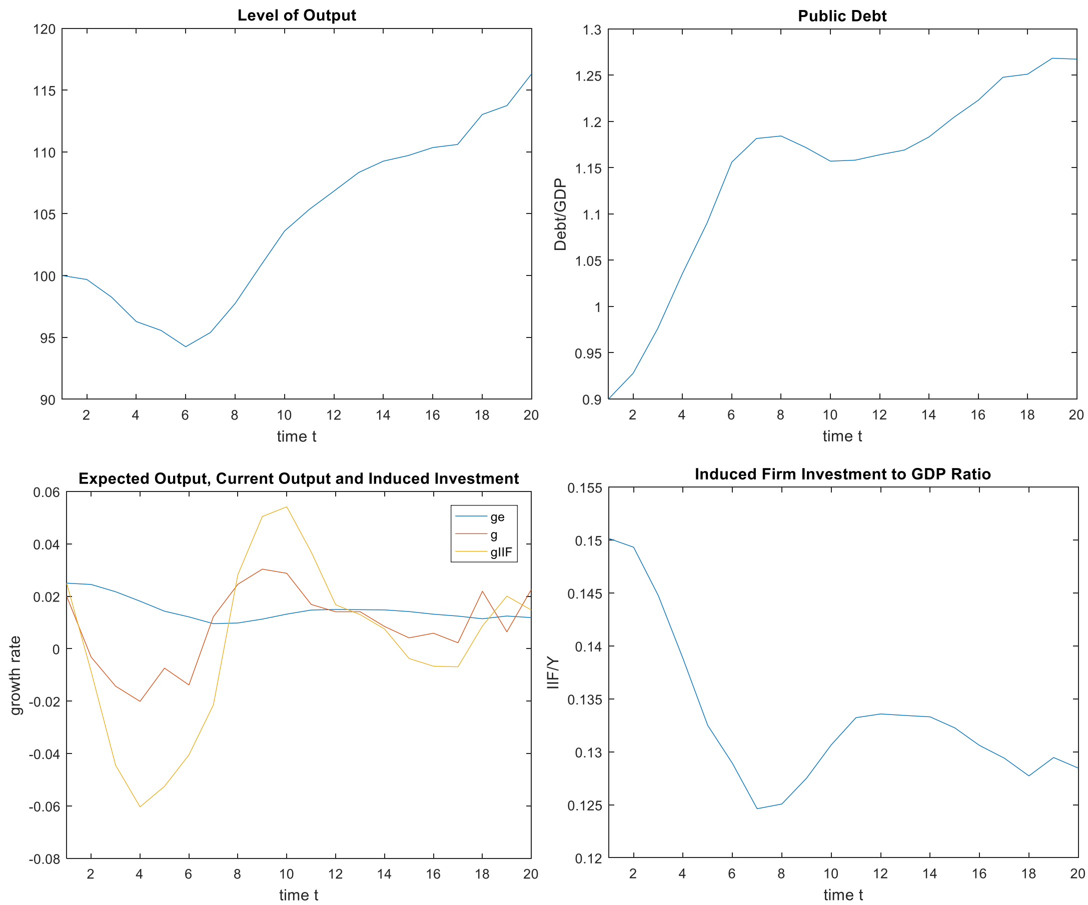
[11] Differently from the adjustment in spending, since the income tax rate is applied to GDP as a whole as well as to the stock of past debt, the increase in the tax rate is smaller in percentage points than the required revenue adjustment in percentage of GDP. In our example, the $\Delta\ \text{revenue} = 0.5\ \%\ \text{of GDP}$, will result in an adjustment of the tax rate (which is the same for all types of income for simplicity) of less than 0.005.

This type of rule means past GDP growth is used to estimate current GDP growth, but the adjustment to changing GDP growth is slow since $x = 0.10$. Hence, if the primary structural surplus is equal to or higher than 2.5% of potential GDP in a specific year t, government spending is set to grow in $t + 1$ at potential GDP ($gZGt = gstruct_t^e$) and direct taxation rates remain at the same level as in year t.

5. Some simulation results

Starting with Country 1, Graph 2 shows that the attempts to bring down the debt to GDP ratio by increasing the structural balance rule during the external shock are ineffective and lead to a significant GDP loss. Only when the economy grows again does the debt to GDP ratio stabilize. Although the structural balance is greatly increased, as shown in Graph 3 below, the current nominal balance remains low until the economy grows again. Graph 2 also illustrates how the growth rate of induced investment has a marked cyclical behavior, exacerbating changes in output growth as investment adjusts to changes in the expected growth rate. In turn, this affects the induced firm investment to GDP ratio, which suffers a precipitous decrease in the first few periods, recovering only partially when the growth of output resumes.

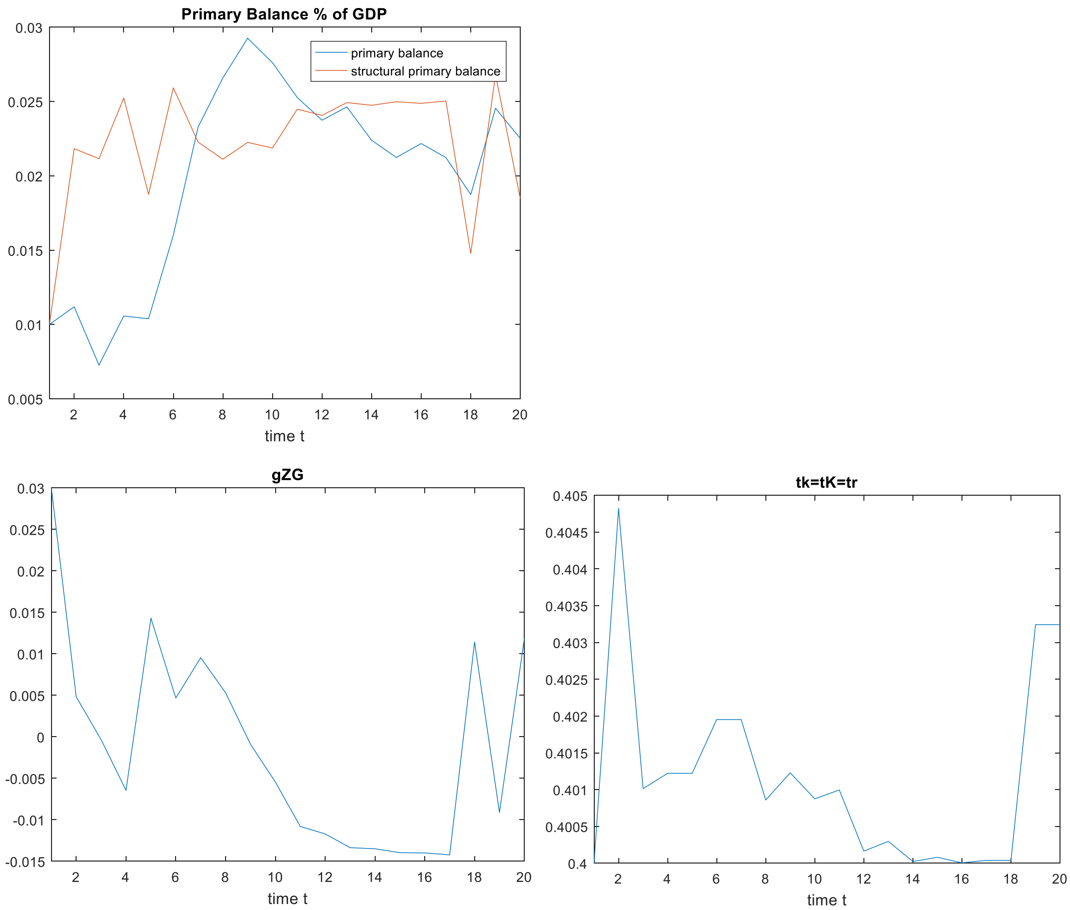
Graph 2 | Output, Induced Investment, and Public Debt – Country 1 (Brazil)



Source: own calculations.

The fiscal policy rule adopted is in great part responsible for the result illustrated in Graph 3 below. After an initial cut in government spending growth gZG in $t = 2$ to $t = 4$ and an increase in initial direct tax rates, the structural balance of 2.5% is achieved in year 4, and spending growth is increased to expected potential growth in $t = 5$, which is estimated to be significantly lower than at the beginning of the simulation. Structural balance is below the 2.5% target in $t = 5$, and austerity resumes in $t = 6$, achieving a structural balance above the target in this year. Government spending once again grows at the potential at $t = 7$, but this is lower than before. However, when the structural balance adjustment process starts, it significantly lowers government spending for several years, turning the latter growth rate to a negative value. This eventually leads to even lower GDP growth and raises the debt to GDP ratio instead of lowering it, which leads to a self-defeating austerity process. At the end of the simulation, government spending growth and taxation alternates between austerity and return to spending growth at potential GDP and lower rates, respectively, as there are attempts to achieve the structural primary surplus target of 2.5% of GDP.

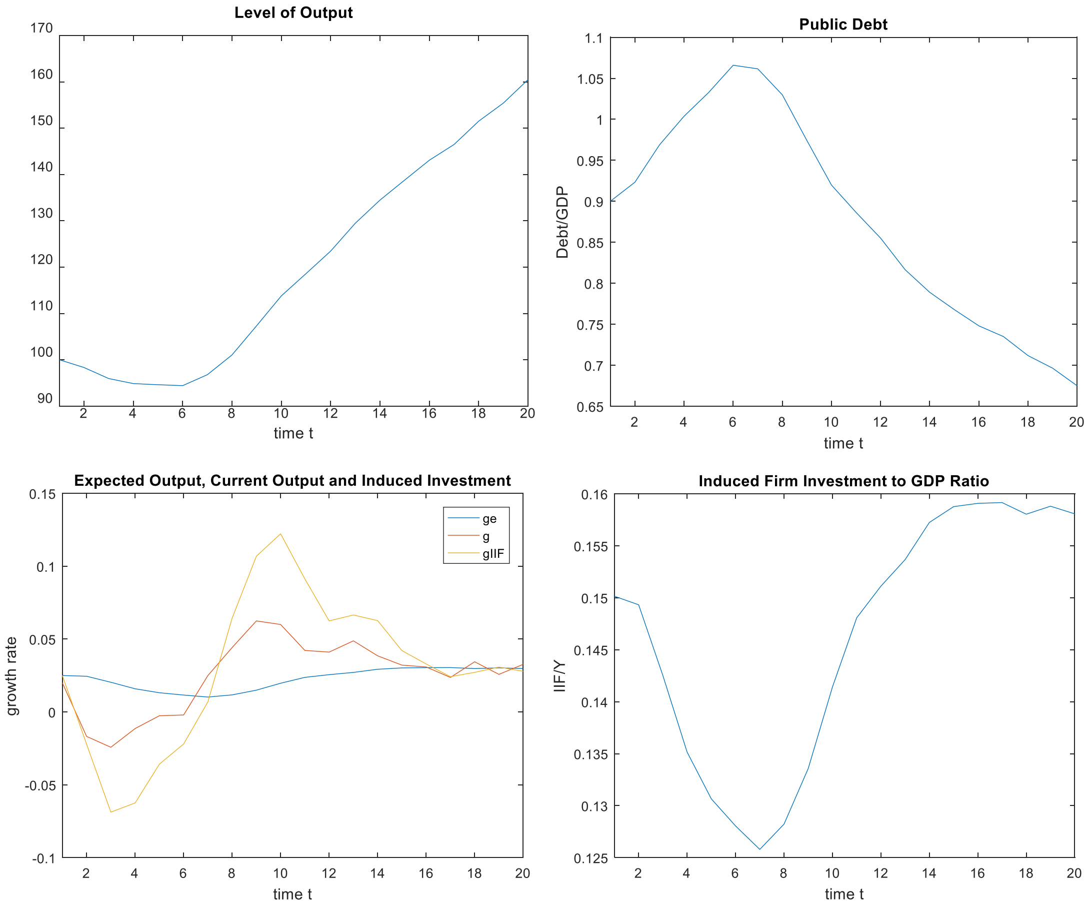
Graph 3 | Fiscal Policy – Country 1 (Brazil)



Source: own calculations.

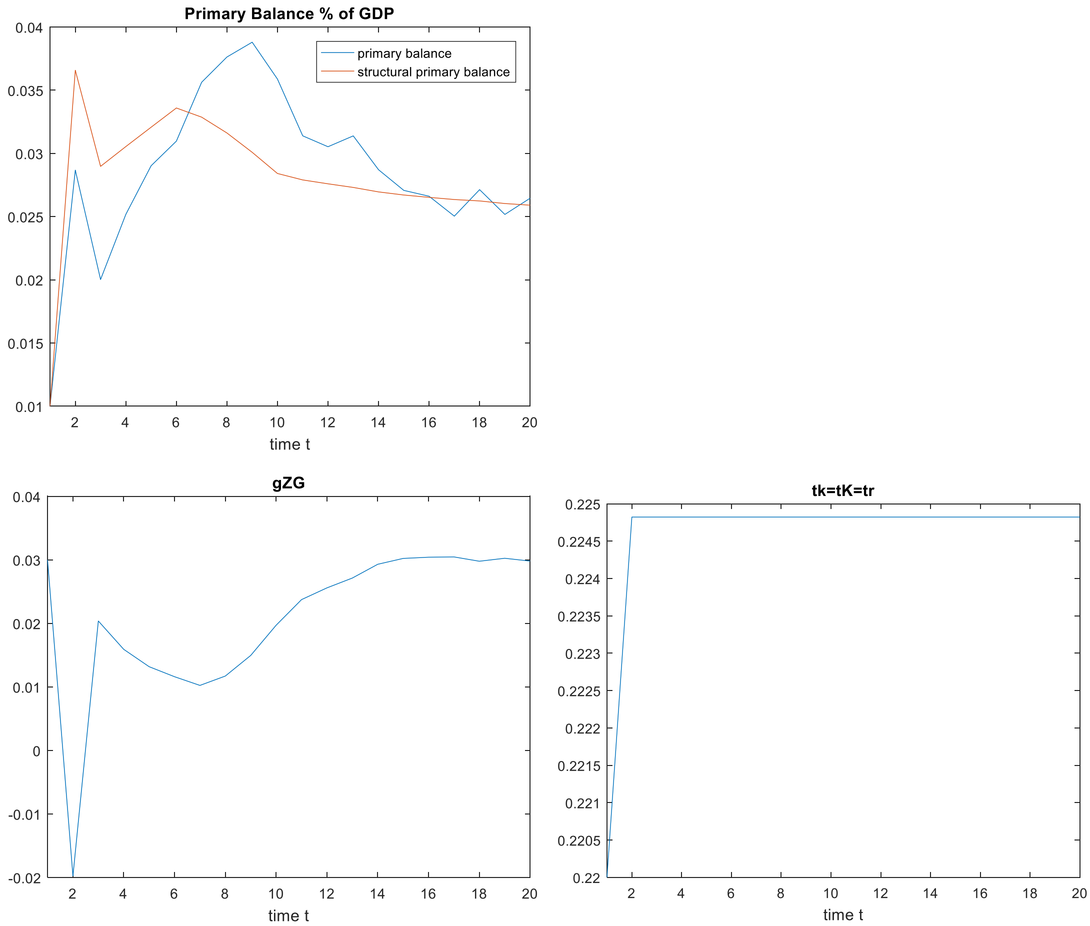
In Graph 4 and Graph 5 below, we see the performance of Country 2. Although the initial shock has a severe effect on the country in the first few years with a significant adverse impact on GDP and the debt to GDP ratio, once growth resumes, the country resumes a path of steady growth and a lowering of the debt to GDP ratio. Even if initially, in the first few periods, just as in the case of Country 1, induced firm investment falls substantially, afterward it behaves somewhat differently, not only recovering its former level but settling at a higher ratio in relation to GDP, as it adapts to a higher trend in the growth rate of the economy. These entirely different results occur with the same structural balance rule.

Graph 4 | Output, Induced Investment, and Public Debt – Country 2 (export-led)



Source: own calculations.

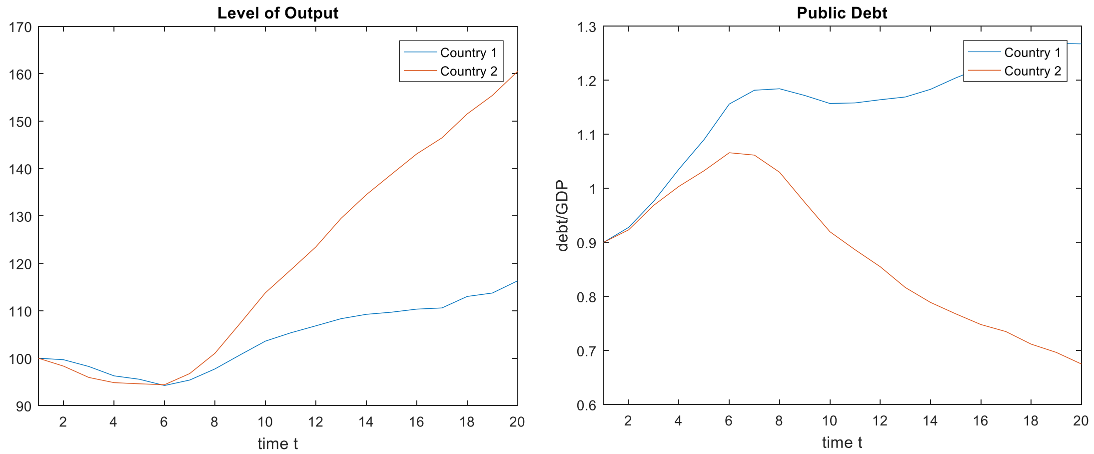
Graph 5 | Fiscal Policy – Country 2 (export-led)



Source: own calculations.

The difference in performance between countries can be better visualized in Graph 6 and Table 4 below. Although both countries experience the shock with a severe contraction, that is accentuated by a fiscal policy that seeks to increase the primary structural balance to 2.5% of GDP from an initial level of 1.0% of GDP that lasts to $t = 6$, the recovery experienced by the export-led country starting at $t = 7$ is a lot more intense.

Graph 6 | Comparing Output and Public Debt in both countries



Source: own calculations.

Table 4 | Comparing Output and Debt to GDP Levels at t=20

	At *t*=20	
	Country 1 (Brazil)	**Country 2 (Export led)**
Level of GDP	116.32	160.51
Debt/GDP (t=20)	126.74%	67.4%

Source: own calculations.

Regarding the debt to GDP ratio, both countries experience a steep rise because of the increase in the real interest rate, but this is more intense in Country 1 than in Country 2. Once GDP growth resumes in both countries (and real interest rates decrease), the ratio stabilizes and, after that, falls, but much more intensely in Country 2. But then, after a few years, the structural balance rule leads to two very different results: a continuously diminishing debt to GDP ratio in Country 2 and a self-defeating fiscal adjustment in Country 1 as debt to GDP rises with continued austerity.

This last result occurs because, in the demand-led growth setting of our model, the structural balance rule has a perverse effect on the growth of potential GDP as it is influenced by the trend rate of observed GDP growth. This feature, combined with the assumption of a relatively high government share of total autonomous demand in Country 1, makes the growth rates of total autonomous demand and GDP very sensitive to government spending growth rate changes. The latter situation leads to more austerity, reinforcing the process and opening space for the self-defeating policy scenario. On the other hand, in the case of Country 2, the relatively low share of government spending in total autonomous demand makes the growth rates of total autonomous demand and GDP less sensitive to changes in the rate of growth of government spending, which facilitates the fiscal adjustment promoted by the structural balance rule. Moreover, note that even when the primary structural balance of 2.5% of GDP is achieved and government spending growth resumes, this occurs at a much lower estimated growth for potential GDP, continuing to reinforce this perverse dynamic process. Indeed, although the latter occurs in both countries, in Country 2 this behavior is restricted to the first part of the simulation, while in Country 1 it persists throughout the entire period of 20 years.[12]

A possible critique of the stylized fiscal rule used above is that it is common practice for fiscal rules, including structural balance rules, to have escape clauses to deal with severe shocks, meaning the rule is too unrealistic to be of use. This choice of stylized rules was intentional, since it illustrates how this type of rule, like most fiscal rules, does not work adequately and has to constantly utilize its escape clause so that it does not lead to perverse results. In an IMF Staff Discussion Note, Eyraud *et al*. (2018, p. 11) point that in the three decades before 2018 fiscal rules were only adhered to approximately 50% of the time, if not considered the use of escape clauses. We argue that this tendency for fiscal rules to be broken or to utilize escape clauses is largely a result of their inadequate design that is illustrated by this need to resort constantly to escape clauses.

6. Concluding remarks

The paper argues that the composition of autonomous expenditure is an important element in explaining the causes of the differing fiscal performances under the same fiscal policy rule. We developed a supermultiplier growth model to show that this performance can differ significantly between countries depending on the government share of total autonomous demand. We suggest that in countries with a relatively high share of government expenditures in total autonomous spending, the results of fiscal rules can lead to significantly lower GDP growth and self-defeating austerity, raising instead of lowering the debt to GDP ratio. In contrast, in countries with relatively low autonomous demand shares of government spending, the same fiscal rule can have a less perverse effect on GDP growth and lead to the intended result of lowering the debt to GDP ratio.

As always is the case with simulation exercises, it is important to evaluate the robustness of our results with respect to different specifications of the model, alternative fiscal policy rules, and economic scenarios. In particular, it would be interesting to go beyond the pure demand-led growth exercise done in this paper. For instance, this could be realized by explicitly incorporating the idea of balance-of-payments constraint along the lines suggested by the literature on the Kaldor-Thirlwall model and considering the influences of external imbalances on aggregate demand growth particularly through the economic policy channel. Another interesting change in the model in the same direction would be the explicit incorporation of labor market dynamics and its repercussions on aggregate demand growth, especially through its effects on income distribution and inflation and the impact of these variables on economic policies. Finally, a significant improvement would be the explicit introduction of natural resources and environmental issues in the model and the investigation of their repercussions on aggregate demand dynamics and economic policies.

[12] One important observation is that the structural balance rule, just as any fiscal balance rule, allows the choice to achieve the result through raising revenue or reducing spending growth in whatever proportion policymakers deem best or possible. It is important to emphasize that this choice leads to significantly different results for the simulations under the same rule. In this respect, see Ligiéro (2021, chap. 3) for simulations with the model illustrating this point.

References

Associação Brasileira das Entidades doso Mercados Financeiro e de Capital (2021); "Estrutura a Termo da Taxa de Juros", https://www.anbima.com.br/pt_br/informar/curvas-de-juros-fechamento.htm, date of consultation: 04/30/2021.

Banco Central do Brasil (2021); "Dívida Bruta do Governo Geral", https://www.bcb.gov.br/estatisticas/tabelasespeciais, date of consultation: 16/01/21.

Cesaratto, S., Serrano, F. and Stirati, A. (2003); "Technical Change, Effective Demand and Employment", *Review of Political Economy*, 15 (1), pp. 33-52.

Ciccone, R. (1986); "Accumulation and Capacity Utilization: Some Critical Considerations on Joan Robinson's Theory of Distribution", *Political Economy: Studies in the Surplus Approach*, 2 (1), pp. 17–36.

Ciccone, R. (1987); "Accumulation, Capacity Utilization and Distribution: A Reply", *Political Economy: Studies in the Surplus Approach*, 3 (1), pp. 97-111.

Eyraud, L.; Debrun, X.; Hodge, A.; Duarte Lledo V. and Pattill, C. (2018); "Second-Generation Fiscal Rules: Balancing Simplicity, Flexibility and Enforceability", IMF, https://www.imf.org/en/Publications/Staff-Discussion-Notes/Issues/2018/04/12/Second-Generation-Fiscal-Rules-Balancing-Simplicity Flexibility-and-Enforceability-45131, date of consultation: 07/28/2018.

Freitas, F. and Christianes, R. (2020); "A Baseline Supermultiplier Model for the Analysis of Fiscal Policy and Government Debt", *Review of Keynesian Economics*, 8 (3), pp. 313-338.

Gandolf, G. (2002); *International Finance and Open-Economy Macroeconomics*, Springer-Verlag.

Haluska, G.; Braga, J. and Summa, R. (2019): "Growth, Investment Share and the Stability of the Sraffian Supermultiplier Model in the United States Economy (1985-2017)", Instituto de Economia da Universidade Federal do Rio de Janeiro - IE- UFRJ, Discussion Paper Nº 024.

Instituto Brasileiro de Geografia e Estatística (2021a); "Sistema Nacional de Contas Trimestrais – SCNT", Tabelas Completas.

Instituto Brasileiro de Geografia e Estatística (2021b); "Sistema de Contas Nacionais – SCN", Tabelas Completas.

Instituto Brasileiro de Geografia e Estatística (2021c); "Inflação IPCA", https://www.ibge.gov.br/explica/inflacao.php, date of consultation: 04/30/21.

Ligiéro, F. (2021); “Regras Fiscais e Teoria Macroeconômica: Origens, Evolução e Visão Crítica”, unpublished MS Dissertation, Instituto de Economia da Universidade Federal do Rio de Janeiro.

Miyazawa, K. (1976); “Input-Output Analysis and the Structure of Income Distribution”, *Lecture notes in economics and mathematical systems*, N° 116, Springer-Verlag.

Serrano, F. (1995a); “Long Period Effective Demand and the Sraffian Supermultiplier”, *Contributions to Political Economy*, 14, pp. 67-90.

Serrano, F. (1995b); “The Sraffian Supermultiplier”, unpublished Ph.D. Dissertation, Cambridge University.

Serrano, F., Freitas, F. and Bhering, G. (2019); “The Trouble with Harrod: The Fundamental Instability of the Warranted Rate in the Light of the Sraffian Supermultiplier”, *Metroeconomica*, 70 (2), pp. 263-287.

Souza, J., and Cornélio, F. (2020); “Estoque de Capital Fixo no Brasil: Séries Desagregadas Anuais, Trimestrais e Mensais”, Instituto de Pesquisa Econômica Aplicada (IPEA), IPEA Discussion Paper, Nº 2580.

Appendix 1 – Exogenous variables for temporary shock scenario

T	*gX*	*gIR*	*gIAF*	*gCA*	*r*	*ω*
1	2.00%	2.00%	2.00%	2.00%	4.00%	57.0%
2	-2.00%	2.00%	1.20%	1.50%	4.00%	57.0%
3	-4.00%	1.00%	0.60%	0.75%	4.50%	56.5%
4	0.00%	-2.00%	-1.20%	-1.50%	5.00%	56.0%
5	2.00%	-4.00%	-2.40%	-3.00%	5.50%	55.5%
6	3.00%	-8.00%	-4.80%	-6.00%	6.00%	55.0%
7	4.00%	0.00%	0.00%	0.00%	5.50%	55.5%
8	6.00%	2.00%	1.20%	1.50%	5.00%	56.0%
9	8.00%	4.00%	2.40%	3.00%	4.50%	56.5%
10	6.00%	6.00%	3.60%	4.50%	4.00%	57.0%
11	3.00%	8.00%	4.80%	6.00%	4.00%	57.0%
12	4.00%	6.00%	3.60%	4.50%	4.00%	57.0%
13	6.00%	4.00%	2.40%	3.00%	4.00%	57.0%
14	4.00%	3.00%	1.80%	2.25%	4.00%	57.0%
15	3.50%	2.00%	1.20%	1.50%	4.00%	57.0%
16	3.00%	4.00%	2.40%	3.00%	4.00%	57.0%
17	2.00%	3.00%	1.80%	2.25%	4.00%	57.0%
18	4.00%	4.00%	2.40%	3.00%	4.00%	57.0%
19	2.50%	2.00%	1.20%	1.50%	4.00%	57.0%
20	3.50%	3.50%	3.50%	3.50%	4.00%	57.0%