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Revisiting the Exchange Rate Pass Through: A General Equilibrium Perspective

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Non-Technical Summary

Research Question

How a movement in the exchange rate will affect inflation is one of the main concerns at central banks. A large literature estimates this exchange rate pass-through to prices (ERPT) using reduced-form approaches. These results are heavily used for policy making at Central Banks. We study the usefulness of these empirical measures for monetary policy analysis and decision making, emphasizing two main problems that arise naturally from a general equilibrium perspective. First, while the literature describes a single ERPT measure, in a general equilibrium model the evolution of the exchange rate and prices will differ depending on the shock hitting the economy. Second, in a general equilibrium model the ERPT crucially depends on the expected behavior of monetary policy, but the empirical approaches in the literature cannot account for this; providing a misleading guide for policy makers.

Contribution

We first distinguish between conditional and unconditional ERPT measures. The former refers to the ratio of the percentage change in a price index, relative to that in the NER, that occurs conditional on a given shock. The unconditional or aggregate measure is the analogous ratio obtained from reduced-form methodologies. We show analytically that, under some assumptions, the unconditional ERPT is a weighted average of the conditional ERPTs in the model. Thus, to the extent that the conditional ERPTs are significantly different depending on the shock, the empirical measures will provide a biased assessment of the expected relationship between the NER and prices at any point in time. We also provide an unconditional ERPT measures directly comparable to the empirical literature estimates.

The other main contribution is to study the dependence of ERPT measures on the reaction of monetary policy. The conditional and unconditional ERPTs depend on how monetary policy reacts and is expected to react; but how this fundamental fact is captured in the empirical ERPT estimates is not clear. Thus, the use of reduced-form estimates to forecast the likely dynamics of inflation after a movement in the NER neglects the fact that monetary policy (both actual and expected) will influence the final outcome.

Results

Our analysis is based on two dynamic and stochastic general equilibrium (DSGE) models. The first is a simple small-open-economy model, while the second is a fully-fledged DSGE model with sectoral distinctions, nominal and real rigidities, driven by a wide variety of structural shocks, estimated using Chilean data.

Our results show that the conditional ERPTs for the main drivers of the NER are in fact very different from each other, and that the unconditional measures lie between the conditional ones. This evidence points to the importance of identifying the source of the shock that originates the NER change in discussing the likely effect on prices.

Regarding the dependence of ERPTs on expected monetary policy, we show that both conditional and unconditional ERPT measures can vary significantly depending the expected path for monetary policy rates.

Revisiting the Exchange Rate Pass Through: A General Equilibrium Perspective*

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Abstract

A large literature estimates the exchange rate pass-through to prices (ERPT) using reduced-form approaches; whose results are an important input for analyses at Central Banks. We study the usefulness of these empirical measures for monetary policy analysis and decision making, emphasizing two main problems that arise naturally from a general equilibrium perspective. First, while the literature describes a single ERPT measure, in a general equilibrium model the evolution of the exchange rate and prices will differ depending on the shock hitting the economy. Accordingly, we distinguish between *conditional* and *unconditional* ERPT measures, showing that they can lead to very different interpretations. Second, in a general equilibrium model the ERPT crucially depends on the *expected* behavior of monetary policy, but the empirical approaches in the literature cannot account for this, providing a misleading guide for policy makers. We first use a simple model of a small and open economy to qualitatively show the intuition behind these two critiques. We then highlight the quantitative relevance of these distinctions by means of a DSGE model of a small and open economy with sectoral distinctions, real and nominal rigidities, and a variety of driving forces; estimated using Chilean data.

JEL Classification: E5, F3, F4.

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1 Introduction

The exchange-rate pass-through (ERPT) is a measure of the change in the price of a good (or basket of goods) after a change in the nominal exchange rate (NER), computed at different horizons after the initial movement in the NER. Its estimates are not only a relevant part of the international macroeconomics literature, but for actual monetary policy as well. For instance, when a country is a price taker in the world markets, a change in the nominal exchange rate affects directly the local currency price of the goods bought internationally, and this way importable inflation. It may even affect other sectors of the economy, and for a prolonged period of time if there are propagation mechanisms at play. In the last years this topic has received a renewed interest, particularly since many countries experienced large depreciations after the Tapering announcements by the Fed in 2013.

The relevance of the topic for actual monetary policy making can be seen from three different perspectives. First, in the vast majority of Central Banks one can find studies estimating the ERPT for the particular country. Second, international institutions such as the International Monetary Fund (IMF), the Bank of International Settlements (BIS), and Inter-American Development Bank (IDB), among others, also actively participate in this discussion. For instance, some of the flagship reports of these institutions (such as the World Economic Outlook by the IMF or the Macroeconomic Report by the IDB) frequently include estimates of the ERPT and use them to draw policy recommendations. Moreover, a significant number of papers in this literature come from economists working at these institutions. Finally, it is easy to find references to the ERPT in many Monetary Policy Reports, proceedings from policy meetings, and speeches by board members at many Central Banks.

Policy related institutions and central banks use estimates of ERPTs that are mostly computed using empirical/reduced-form approaches based on vector auto-regressions (VAR) or single equation models.¹ The ERPT measures are generally used for two purposes. The first is to predict the effect that an observed depreciation will have on inflation. The second use is for ex-post analysis, after some time has passed, with the goal of understanding what happened and explain differences, if any, with what was expected to happen. In light of this widespread use, in this paper we question the usefulness of the empirical ERPT measures for these purposes using a general equilibrium framework.

In particular, we highlight two shortcomings of using reduced-form estimates of ERPT for policy analysis, that can be improved by using dynamic and stochastic general equilibrium (DSGE) models. First, empirical ERPT estimates do not control (completely) for the endogeneity of the NER. The evolution of the NER, as well as its relation with other prices, will depend on the shocks hitting the economy.² Second, these empirical estimates do not control for the dependence of ERPT measures on the expected reaction of monetary policy, which can affect them significantly. While the first shortcoming has been discussed recently in the literature, as detailed below, the second has not been analyzed.

We distinguish between *conditional* and *unconditional* (or *aggregate*) ERPT measures. The former refers to the ratio of the percentage change in a price index, relative to that in the NER, that occurs conditional on a given shock. The unconditional or aggregate measure is the analogous ratio obtained from reduced-form methodologies. We show how they relate to each other, we explore their differences and how they depend not only on parameters of the model, but also and more importantly on the reaction assumed for monetary policy.

¹Some examples are [Devereux and Engel \(2002\)](#), [Campa and Goldberg \(2005\)](#), [Campa and Minguez \(2006\)](#), [Choudhri and Hakura \(2006\)](#), [Ca' Zorzi et al. \(2007\)](#), [Gopinath et al. \(2010\)](#), among many others. [Burstein and Gopinath \(2014\)](#) and [Aron et al. \(2014\)](#) provide extensive surveys of this literature. In the rest of the paper, we use the terms “reduced-form” and “empirical” interchangeably to refer to this literature.

²The empirical literature tries to overcome this endogeneity by isolating “exogenous” movements in the NER but, as we will argue, not all surprise movements in the NER are alike.

Our analysis is based on two dynamic and stochastic general equilibrium (DSGE) models. The first is a simple small-open-economy model, with traded and non-traded goods and price rigidities. This model allows to grasp the intuition behind the two shortcomings of the empirical literature that we highlight, but is not built to talk about its quantitative relevance. To that end, we then set up a fully-fledge DSGE model with sectoral distinctions, nominal and real rigidities, driven by a wide variety of structural shocks. We estimate it using a Bayesian approach with quarterly Chilean data from 2001 to 2016.³

Our first contribution is to study the relationship between conditional and unconditional ERPTs. We first show analytically that, under certain assumptions in the context of linear, dynamic and stochastic models, the unconditional ERPT obtained using a VAR is a weighted average of the conditional ERPTs in the model. Thus, to the extent that the conditional ERPTs are significantly different depending on the shock, the empirical measures will provide a biased assessment of the expected relationship between the NER and prices at any point in time. In general, using the unconditional ERPT will systematically miss the expected evolution of the NER and prices.

Our second contribution is to define unconditional ERPT measures directly comparable to the empirical literature estimates. In general, the mapping between unconditional and conditional ERPTs cannot be obtained algebraically, so we define two measures that can be computed for any model to mimic what an econometrician from the empirical literature would obtain if the general equilibrium model was the true data generating process.

Our third contribution is to study the dependence of ERPT measures on the reaction of monetary policy. As any endogenous variable, the conditional and unconditional ERPTs depend on how monetary policy reacts and is expected to react. How this fundamental fact is captured in the empirical ERPT estimates is not clear. It might be argued that in these estimates it is implicitly assumed that monetary policy follows a policy rule that captures the “average” behavior followed by the central bank, during the sample analyzed. However, as there is no explicit description of this rule, it is hard to know what the central bank is assumed to be doing (and expected to do) in the estimated ERPT coefficient. Thus, the use of reduced-form estimates as a way to forecast the likely dynamics of inflation after a movement in the NER neglects the fact that monetary policy (both actual and expected) will influence the final outcome. With this in mind, one could instead compute several ERPT measures, one for each alternative expected path for monetary policy that a central bank might consider.

Our results show that the conditional ERPTs for the main drivers of the NER are in fact very different from each other, and that the unconditional measures lie between the conditional ones. The analysis is done for 3 different price indexes; the consumer price index (CPI), a tradable and a non-tradable price indexes. In the quantitative model, the two main drivers of the NER are found to be a common trend in international prices and shocks affecting the interest rate parity condition. The conditional ERPT each of them generate are quantitatively different, varying depending on the time period and the price index that is being considered. At the same time, the unconditional ERPT lies between the conditional ones, and are comparable with empirical estimates. Overall, this evidence points to the importance of identifying the source of the shock that originates the NER change in discussing the likely effect on prices.

³Chile is an interesting case of study for several reasons. First, it is a large commodity exporter with a high degree of financial capital mobility, which makes it relatively easy to identify the sources of foreign shocks. Second, since 2001 the Central Bank has followed a flexible inflation targeting strategy, that has been stable during the sample and is considered as one of the success cases of inflation targeting, particularly in Latin America. This greatly facilitates the estimation of a DSGE model, without having to deal with possible shifts in the monetary policy framework. Finally, the exchange rate has moved freely most of the time during this sample, which is quite useful to show how diverse shocks may affect the NER. Nonetheless, the main points made in the paper are conceptually quite general, going beyond the particular country chosen for the estimation.

The results concerning the dependence of ERPTs on monetary policy is show to be model dependent. In principle, it is not clear how the ERPT will differ under alternative policy paths, since a more dovish policy will induce a higher inflation and a larger nominal depreciation. In our simple DSGE model, the effect under alternative policy paths is stronger after a shock that affects the interest rate parity condition than after a shock to external prices. The opposite is true in our fully-fledged DSGE model for Chile. While it remains an open question which conditional ERPT is more sensitive for other countries and other models, this emphasizes the importance of analyzing these issues with model that can properly account for the observed dynamics.⁴

In terms of the related literature, [Shambaugh \(2008\)](#) and [Forbes et al. \(2015\)](#) compute different ERPTs depending on shocks using VAR models. They use alternative identification assumptions to estimate how several shocks might generate different ERPTs; in the same spirit as our definition of conditional pass-through. Our work deepens their analysis in two ways. First, these studies do not show how these conditional ERPT measures compare with unconditional ones; a comparison that we explicitly perform to understand the bias that might be generated by relying on unconditional ERPTs. Second, they use structural VAR models whose identified shocks are still too general as compared to the shocks in a DSGE model.⁵ Our approach can then provide a relatively more precise description of the relevant conditional ERPTs.

Two related papers using DSGEs are those by [Bouakez and Rebei \(2008\)](#) and [Corsetti et al. \(2008\)](#). The work by [Bouakez and Rebei \(2008\)](#) is, to the best of our knowledge, the only one that uses an estimated DSGE to compute conditional ERPTs (estimating the model with Canadian data) and that also provides a measure that would qualify as unconditional ERPT. Our paper differs from theirs since it provides an unconditional ERPT measure that is directly comparable to the methodology implemented in the empirical literature, and it also analyzes the specific relationship between the measures obtained in the reduced-form approaches with the dynamics implied by a DSGE model. Moreover, our estimated DSGE model has a richer sectoral structure, allowing to characterize not only the ERPT for total inflation, but also that for different prices such as tradables and non-tradables. [Corsetti et al. \(2008\)](#) explore the structural determinants of an ERPT to import prices from a DSGE perspective and assess possible biases in single-equation empirical methodologies. While our paper shares common points with this study, we distinguish between conditional and unconditional ERPTs and provide a quantitative evaluation of the biases. Still, none of these studies explore the second shortcoming we highlight regarding the expected monetary policy.

The relationship between monetary policy and the ERPT has been the topic of several studies, but none has analyzed explicitly how alternative expected paths of the monetary rate affects the ERPT, which is a crucial input for policy makers. For instance, [Taylor \(2000\)](#), [Gagnon and Ihrig \(2004\)](#) and [Devereux et al. \(2004\)](#) use dynamic general equilibrium models to see how monetary policy can alter the ERPT, proposing that a greater focus on inflation stabilization can provide an explanation to why the empirical measures of ERPT seem to have declined over time in many countries. Others have analyzed how monetary policy should be different depending on structural characteristics associated with the ERPT, such as the currency in which international prices are set, the degree of nominal

⁴This analysis is based on the comparison between following an estimated Taylor-type rule after a given depreciation and more dovish alternatives. This exercise tries to mimic what would happen if a policy maker is presented with an estimated ERPT coefficient that is relatively low and convinces itself that the likely effect on inflation will be small, deciding not to change the policy stance.

⁵[Shambaugh \(2008\)](#) uses long-run restrictions and identifies shocks such as relative demand, relative supply, nominal, among others. In contrast, with our DSGE model, we can identify a variety of shocks that fall into each of these categories, each of them generating different conditional ERPTs. In the case of [Forbes et al. \(2015\)](#), shocks are identified by sign restrictions, which does not take into account that shocks that imply very different dynamics can have the same sign responses. In fact, in our estimated model the two main drivers of NER movements generate the same sign for impulse responses for most observables, but they imply significantly different ERPTs.

rigidities, among others. Some examples are [Devereux et al. \(2006\)](#), [Engel \(2009\)](#), [Devereux and Yetman \(2010\)](#), and [Corsetti et al. \(2010\)](#). The point we want to stress, although related to these previous papers, is however different: the choice of the expected policy path can influence significantly the realized ERPT; an issue that is generally omitted in policy discussions.

Finally, this paper relates to the extensive literature comparing DSGEs and VARs in terms of their usefulness for different types of analyses. As discussed in [Giacomini \(2013\)](#) there are several reasons why the mapping between a DSGE and a VAR can be broken, making the use of DSGEs beneficial in some cases and of VARs in others. On one hand, it is a general belief that VARs perform better in forecasting variables than DSGEs because they imply less restrictions as can be seen in [Schorfheide \(2000\)](#)⁶. On the other hand, DSGEs are better suited to understand the intuition and mechanisms behind economic movements, since these can be tracked to the original structural shocks as is the case of this paper. In addition, as our analysis of the dependence of ERPTs to the monetary policy reaction highlights, DSGEs are better suited to analyze counterfactual scenarios and to understand which parameter or mechanism is critical for a given result.

The rest of the paper is organized as follows. Section 2 describes the empirical strategies used in the literature and their relationship with DSGE models. The analysis based on a simple model is presented in Section 3. The quantitative DSGE model and the ERPT analysis based on it are included in Section 4. Conclusions are discussed in Section 5.

2 The Empirical Approach to ERPT and DSGE Models

In this section we first describe two methodologies generally used in the reduced-form literature to estimate the ERPT: single-equation and VAR models. We then use a general linearized DSGE model to introduce the concept of conditional ERPT. Finally, we discuss the relationship between conditional ERPTs from DSGE models and the measure obtained using a VAR approach.

2.1 The Empirical Approach

The two approaches most commonly used by the empirical literature are single-equation models and VARs. In the first the estimated model takes the form,

$$\pi_t^j = \alpha + \sum_{j=0}^K \beta_j \pi_{t-j}^S + \gamma c_t + v_t, \quad (1)$$

where π_t^j denotes the log-difference in the price of a good (or basket of goods) j , π_t^S is the log-difference of the NER, c_t is a vector of controls and v_t is an error term. The parameters α , β_j , and γ are generally estimated by OLS, and the ERPT h periods after the movement in the NER is computed as $\sum_{j=0}^h \beta_j$; i.e. the percentage change in the price of good j generated by a 1% permanent change in the NER.

The VAR strategy specifies a model for the vector of stationary variables x_t that includes π_t^S , π_t^j , as well as other control variables (both of domestic and foreign origin). The reduced-form VAR(p) model is,

$$x_t = \Phi_1 x_{t-1} + \dots + \Phi_p x_{t-p} + u_t, \quad (2)$$

where Φ_j for $j = 1, \dots, p$ are matrices to be estimated, and u_t is a vector of i.i.d. reduced-form shocks, with zero mean and variance-covariance matrix Ω . Associated with u_t , the “structural” disturbances

⁶This view has been challenged by several authors specially after DSGEs have included features that increase their fit to data starting with [Smets and Wouters \(2003\)](#).

w_t are defined as,

$$u_t = Pw_t, \quad (3)$$

where P satisfies $\Omega = PP'$, assuming the variance of w_t equals the identity matrix. In the empirical ERPT literature P is assumed to be lower triangular, obtained from the Cholesky decomposition of Ω , and the ERPT h periods ahead is defined as.

$$ERPT_{\pi^j}^V(h) \equiv \frac{CIRF_{\pi^j, \pi^S}^V(h)}{CIRF_{\pi^S, \pi^S}^V(h)}, \quad (4)$$

where $CIRF_{k,i}^V(h)$ is the cumulative impulse-response of variable k , after a shock in the position associated with variable i , h periods after the shock. In other words, the ERPT is the ratio of the cumulative percentage change in the price, relative to that in the NER, originated by the shock associated with the NER in the Cholesky order.⁷

While both approaches can be found in the literature, here we use the VAR as a benchmark for several reasons. First, in the most recent papers the VAR approach is generally preferred. Second, the ERPT obtained from (1) assumes that after the NER moves, it stays in that value forever. In contrast, the measure (4) allows for richer dynamics in the NER after the initial change. Third, the OLS estimates from (1) will likely be biased, as most of the variables generally included in the right-hand side are endogenous. The VAR attempts to solve this problem by including lags of all variables, and by means of the identification strategy, as long as the Cholesky decomposition is correct.⁸ Finally, the VAR model might, in principle, be an appropriate representation of the true multivariate model (as we will discuss momentarily), but this is not generally true for single-equation models.

2.2 DSGE Models and Conditional ERPT

The linearized solution of a DSGE model takes the form,

$$y_t = Fy_{t-1} + Qe_t, \quad (5)$$

where y_t is a vector of variables in the model (exogenous and endogenous, predetermined or not), e_t is a vector of *i.i.d.* structural shocks, with mean zero and variance equal to the identity matrix, and the matrices F and Q are non-algebraic functions of the deep parameters in the model.⁹

Using the solution, the ERPT conditional on the shock e^i for the price of good j is defined as,

$$CERPT_{\pi^j, i}^M(h) \equiv \frac{CIRF_{\pi^j, e^i}^M(h)}{CIRF_{\pi^S, e^i}^M(h)}, \quad (6)$$

which is analogous to the definition of $ERPT_{\pi^j}^V(h)$ in (4), with the difference that the response is computed after the shock e_t^i , and we can compute one for each shock in the vector e_t . This means that the conditional ERPT is the ratio between the cumulative percentage change in the inflation of

⁷In general, it is assumed that π_t^S is ordered before π_t^j in the vector x_t . In addition, if the vector x_t contains foreign variables and the country is assumed to be small relative to the rest of the world, these variables are ordered first in x_t and the related matrices Φ_j are assumed to have a block of zeros to prevent feedback from domestic variables to foreign ones at any lag.

⁸We will describe in the next subsection how that assumption will generally not hold if a DSGE model is the true data generating process. But at least the VAR methodology attempts to deal with the endogeneity issue, while the single-equation OLS based approach does not.

⁹This solution can be obtained by several methods after linearizing the non-linear equilibrium conditions of the model around the non-stochastic steady state, and can be implemented in different packages, such as Dynare.

price j , relative to the cumulative change in the NER, originated by shock e^i .

2.3 The Relationship Between VAR- and DSGE-based ERPT

We want to explore the relationship between $ERPT_{\pi^j}^V(h)$ and $CERPT_{\pi^j,i}^M(h)$, in order to construct a measure of unconditional ERPT from the DSGE model that is comparable to $ERPT_{\pi^j}^V(h)$. Relevant for this discussion is the work of [Ravenna \(2007\)](#), who explores conditions under which the dynamics of a subset of variables in the DSGE model can be represented with a finite-order VAR model. The general message is that it is not obvious that a DSGE model will meet these requirements, implying that the relationship we wish to find can only be obtained analytically for specific cases.¹⁰

In [Appendix A.1](#) we show that, if the assumptions for the existence of a finite VAR representation of the DSGE model hold, and if π_t^S is ordered first in the VAR, the following relationship holds

$$ERPT_{\pi^j}^V(h) = \sum_{s=1}^{n_e} CERPT_{\pi^j,s}^M(h) \omega_s(h), \quad (7)$$

where n_e is the number of shocks in the vector e_t and $\omega_s(h)$ are weights associated with each shock. In other words, the ERPT obtained from the VAR is a weighted sum of the conditional ERPTs in the DSGE model. For $h = 0$ the weight $\omega_s(0)$ corresponds to the fraction of the forecast-error variance of the NER, at horizon $h = 0$, explained by the shock s . For $h > 0$ the weight $\omega_s(h)$ is equal to $\omega_s(0)$ adjusted by the change in the response of the NER at horizon $h > 0$ relative to the response at $h = 0$.¹¹ In simpler terms, the weights depend on the relative importance that each shock has in explaining the fluctuations in the NER. Moreover, the relative importance of the particular shock in accounting for the dynamics of inflation is not relevant for its weight in the unconditional ERPT.

The relationship (7) is an important result because it implies that, to the extent that the conditional ERPTs are different, predicting the effect on a price of any movement of the NER with the unconditional measure will almost surely be inappropriate. It will only give a correct assessment of the likely dynamics of inflation if the combination of shocks hitting the economy in a given moment is equal to the weights implicit in the VAR-based ERPT. But in the context of shocks with a continuous support, this event has zero probability. As we will see in the next sections the conditional ERPTs are indeed very different and so this is actually an important disadvantage of using unconditional ERPTs.

The conditions behind (7) may not hold in general DSGE models. Thus, we propose two alternatives to compute the unconditional ERPT. The first one assumes that the relationship in (7) holds in general. We label this as $UERPT_{\pi^j}^M(h) \equiv \sum_{s=1}^{n_e} CERPT_{\pi^j,s}^M(h) \omega_s(h)$, where $CERPT_{\pi^j,i}^M(h)$ is computed as in (6), and $\omega_s(h)$ is analogous to the one in (7).

The second measure of unconditional ERPT answers the following question: what would be the ERPT that someone using the empirical VAR approach would estimate if she has an infinite sample of the variables commonly used in that literature, generated by the DSGE model? We call this alternative unconditional ERPT using a Population VAR, labeled as $UERPT_{\pi^j}^{PV}(h)$; which is analogous to (4) but the matrices Φ_j and Ω are obtained from the population (i.e. unconditional) moments computed from the solution of the DSGE model.¹²

In conclusion, for any particular DSGE model, we have two unconditional ERPTs to compare

¹⁰ A related issue is analyzed by [Fernández-Villaverde et al. \(2007\)](#), showing conditions under which the shocks identified in a VAR for a subset of the variables in a DSGE can capture the same shocks featured in the DSGE model. However, as the empirical VAR literature of ERPT does not claim that it is identifying any particular shock that can be interpreted from a DSGE model, this aspect is not as relevant for our discussion.

¹¹ See [Appendix A.1](#) for the precise expression for $\omega_s(h)$.

¹² [Appendix A.2](#) shows how this is computed.

with the conditional ones, in order to assess their differences. In the following sections we apply these measures to both a simple and a quantitative DSGE model.

3 A Simple DSGE Model

In this section we develop a simple DSGE model to show the importance of differentiating between conditional and unconditional ERPT, as well as of accounting for the expected paths of monetary policy. The model is based on [Schmitt-Grohé and Uribe \(2017, sec. 9.16\)](#), extended to include a Taylor rule for the interest rate, indexation and external inflation.

3.1 Description of the Model

The model is relatively small and has only the necessary ingredients to highlight the differences in ERPTs that we want to show. It features three shocks (world interest rate, external inflation and monetary policy) to show the differences between conditional and unconditional ERPTs,¹³ and it features two sectors (tradable, T , and non-tradable, N) to show differences between ERPT in different prices. Monetary policy sets the short-term interest rate following a Taylor rule in the baseline case and this assumption is temporarily relaxed later on to evaluate effects of alternative policy paths. Finally, it includes Calvo pricing in sector N with indexation to past inflation, for its importance in the transmission of changes in the exchange rate to internal prices. In what follows we describe the different agents in the model, while Appendix [B](#) presents all the equilibrium conditions and the computation of the steady state.

3.1.1 Households

There is a representative household that consumes, works and saves. Her goal is to maximize,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \xi \frac{h_t^{1+\varphi}}{1+\varphi} \right\}$$

where C_t is consumption and h_t are hours worked, β is the discount factor, σ is the risk aversion parameter, φ is the inverse of the Frish elasticity of labor supply and ξ is a scale parameter. Her budget constraint is

$$P_t C_t + S_t B_t^* + B_t = h_t W_t + S_t R_{t-1}^* B_{t-1}^* + R_{t-1} B_{t-1} + \Pi_t.$$

Here P_t is the price of the consumption good, S_t is the exchange rate, B_t^* is the amount of external bonds bought by the household in period t , B_t the analogous for local bonds bought by the household in t , W_t is the wage, R_t^* is the external interest rate, R_t is the domestic interest rate, and Π_t collects all the profits from the firms in the economy, since households are the owners of firms.

The consumption good is a composite of tradable consumption, C_t^T , and non-tradable consumption, C_t^N . Additionally, non-tradable consumption is an aggregate of non-tradable varieties, $C_t^N(i)$. These technologies are described by,

$$\begin{aligned} C_t &= \left[\gamma^{1/\varrho} (C_t^N)^{\frac{\varrho-1}{\varrho}} + (1-\gamma)^{1/\varrho} (C_t^T)^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}} \\ C_t^N &= \left[\int_0^1 (C_t^N(i))^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \end{aligned}$$

¹³Those shocks were chosen because of their importance in the larger model of the next section.

where γ is the share of N in total consumption, ϱ is the elasticity of substitution between C_t^N and C_t^T , and ϵ is the elasticity of substitution between the varieties $i \in [0, 1]$ of non-tradables. From the problem of choosing the minimum expenditure to get the consumption good, we obtain the definition of the consumer price level as,

$$P_t = [(1 - \gamma)(P_t^T)^{1-\varrho} + \gamma(P_t^N)^{1-\varrho}]^{\frac{1}{1-\varrho}}$$

where P_t^T is the local price of the tradable good and P_t^N is a price index for the non-tradable composite.

3.1.2 Firms

There are two sectors, tradable and non-tradable. The former is assumed to have a fixed endowment, Y^T , each period with a local price $P_t^T = S_t P_t^{T,*}$, where $P_t^{T,*}$ is the foreign price of the tradable good. In contrast, in the non-tradable sector, each firm $j \in [0, 1]$ produces using labor with the technology

$$Y_t(j) = h_t(j)^\alpha,$$

where $Y_t(j)$ is the production of firm j , $h_t(j)$ is the hours hired and $\alpha \in (0, 1]$ is a parameter. Firm j faces a downward sloping demand given by:

$$Y_t(j)^N = \left(\frac{P_t^N(j)}{P_t^N} \right)^{-\epsilon} Y_t^N$$

where ϵ is the elasticity of substitution among varieties, $P_t^N(j)$ is the price of variety j in the N sector and Y_t^N is non-tradable composite. They choose prices a la Calvo, where the probability of choosing prices each period is $1 - \theta$. In the periods that firms don't choose prices optimally, they update their prices using a combination of past inflation, π_{t-1} and the inflation target, $\bar{\pi}$:

$$\pi_{t-1}^\zeta \bar{\pi}^{1-\zeta}$$

where $\zeta \in [0, 1]$. Note that all prices that are not chosen optimally are indexed either statically to $\bar{\pi}$, or dynamically to π_{t-1} . The final dynamic indexation in the model is given by $\theta\zeta$, since it is the fraction indexed to past inflation, ζ , among the prices that are not chosen optimally, θ . Note also that in the long-run indexation is complete, in the sense that all prices will grow at the same rate $\bar{\pi}$. This eliminates the welfare cost of price dispersion in steady state (and in a first-order approximation).

3.1.3 Monetary Policy

We assume a simple Taylor rule for the domestic interest rate:

$$\left(\frac{R_t}{R} \right) = \left(\frac{\pi_t}{\bar{\pi}} \right)^{\alpha_\pi} \left(\frac{GDP_t}{GDP} \right)^{\alpha_y} \exp(e_t^m)$$

where the variables without a time subscript are steady state values, GDP_t is gross domestic product (see the appendix for a definition) and e_t^m is the monetary shock, assumed to be *i.i.d.*.

3.1.4 Foreign Sector

The rest of the world provides the external price of the tradable output, $P_t^{T,*}$ and the external interest rate, R_t^* . For the first, we assume that foreign inflation, $\pi_t^* \equiv P_t^{T,*}/P_{t-1}^{T,*}$, follows an exogenous process.

For the second, we assume that the external interest rate relevant for the country, R_t^* is given by

$$R_t^* = R_t^W + \phi_B \left(\exp(\bar{b} - B_t^*/P_t^{T,*}) - 1 \right)$$

where R_t^W is the risk-free external interest rate, which follows an exogenous process and $\phi_B, \bar{b} > 0$ are parameters. This equations is the closing device of the model.

3.1.5 Exogenous Processes and Parametrization

The model includes 3 shocks: the monetary policy shock, ϵ_t^m , foreign inflation, π_t^* , and the risk-free external interest rate, R_t^W . It is assumed that each one of these shocks follows a process

$$\log(x_t/x) = \rho_x \log(x_{t-1}/x) + u_t^x,$$

for $x_t = \{\epsilon_t^m, \pi_t^*, R_t^W\}$ and u_t^x is *i.i.d.*. For simplicity, we assume $\rho_x = 0.5$ for $x = \{\pi^*, R^W\}$ and $\rho_{\epsilon^m} = 0$, which is the regular case used in the literature. We allow for the monetary shock to have a positive autocorrelation coefficient later on to highlight the connection between different expected monetary paths and ERPTs. Table 1 shows the parametrization used, which closely follows [Schmitt-Grohé and Uribe \(2017, sec. 9.16\)](#). In the baseline parametrization, we set the indexation parameter to zero, to later explore the role of different values for ζ .

Table 1: Parametrization Simple Model

Parameter	Value	Description
β	1.0316^{-1}	Discount factor
σ	2	Risk aversion
φ	0.5	Inverse Frisch elasticity
ϱ	0.5	Elasticity of substitution between C^T and C^N
γ	0.74	Share of C^N in C
α	0.75	Labor share in N
ϵ	6	Elasticity of substitution across varieties N
θ	0.7	Probability of no price change in N sector
ζ	0	Indexation to past inflation in N sector
α_π	1.5	Taylor rule parameter of π
α_y	0.5/4	Taylor rule parameter of GDP
ϕ_B	0.0000335	Parameter of debt-elastic interest rate
$\bar{\pi}$	$1.03^{1/4}$	Inflation target
pT	1	Relative price of tradables in steady state
h	0.5	Hours worked in steady state
s^{tb}	0.05	Share of trade balance in GDP in steady state

Notes: The source of all parameters is [Schmitt-Grohé and Uribe \(2017, sec. 9.16\)](#), except the ones in the Taylor rule and the steady state values. For the ones in the Taylor rule it is [Taylor \(1993\)](#) and the steady state values are normalizations. s^{tb} was chosen such that the country is a net debtor in steady state.

3.2 Conditional vs. Unconditional ERPTs

In this section we show how even in this simple model there are significant differences among the conditional ERPTs, depending on the shock that is hitting the economy and also on the price considered. Note first that, by construction, the reaction of tradable inflation and the nominal exchange rate

depreciation is the same for the monetary shock and the shock to the external interest rate, implying a conditional ERPT for these shocks equal to one at all horizons. This is because prices in the tradable sector are given by the foreign price of the tradable good, which is exogenous, times the NER. Also note that since the real exchange rate and all relative prices are stationary in the model, these shocks will also have a conditional ERPT of one in the long run for non-tradable and total prices. In contrast, this is not the case for the shock to foreign-inflation, which does not require a complete ERPT to any domestic price, at any horizon.

To understand the propagation of the different shocks, we first present the impulse-response analysis. A positive change in the external interest rate, showed in figure 1, causes two effects: a negative income effect (because this economy is assumed to be a net debtor), and an intertemporal substitution effect, increasing the incentives to save today. Both of them decrease current demand of both goods, while increasing labor supply at the same time. The drop in the demand for non-tradables, as well as the increase in labor supply, tend to decrease the relative price of these goods, leading to a real depreciation.¹⁴ Due to sticky prices, the nominal exchange rate also increases. Inflation rises for both types of goods and, as a result, the policy rate increases.¹⁵

A negative shock to external inflation, showed in figure 2, affects the economy through several channels.¹⁶ In principle, this shock should affect export-related income, generating a wealth effect. However, as the domestic price of tradables is fully flexible, *ceteris paribus*, the relevant relative price (the price of exports over that of imports) does not change; so this channel is not active in this simple model.¹⁷ Another channel is due to the fact that foreign bonds are denominated in dollars: an unexpected drop in foreign prices will increase, *ceteris paribus*, the burden of interest payments from external debt in domestic currency units, generating a negative wealth effect. This channel tends to contract aggregate demand, which reduces consumption of both goods and increases labor supply. Since the non-tradable sector has to clear, its relative price falls. Both a nominal and a real depreciation materialize, inflation rises for both types of goods and the policy rate increases. While qualitatively these effects are analogous to those originated by a rise in the world interest rate, there is an attenuation effect that happens due to the drop in foreign inflation, which leads to a smaller conditional ERPT.

Finally, a negative shock to the policy rule, showed in figure 3 generates a drop in the nominal interest rate for a given value of inflation and output. This causes an intertemporal substitution effect towards current consumption.¹⁸ The higher demand of non-tradables causes an increase in its relative price as well as a rise in its output. This leads to both a real and nominal depreciation, which increases inflation.

We now turn to the conditional ERPTs which, as can be seen in figure 4, significantly differ depending on the shock. First note that, as expected, the ERPTs of tradable prices is in general much higher than of non-tradable, since the former is not subject to price rigidities. For tradable prices, as discussed at the beginning of the section, the conditional ERPT given either a monetary or foreign

¹⁴The effect on the equilibrium consumption (and output) of non-tradables depends on which of the two changes (drop in the demand, or increase in supply) dominates. Given the chosen parametrization, in the short run output contracts, and then it increases above the steady state. In contrast, tradable consumption drops after the shock and converges to the steady state from below.

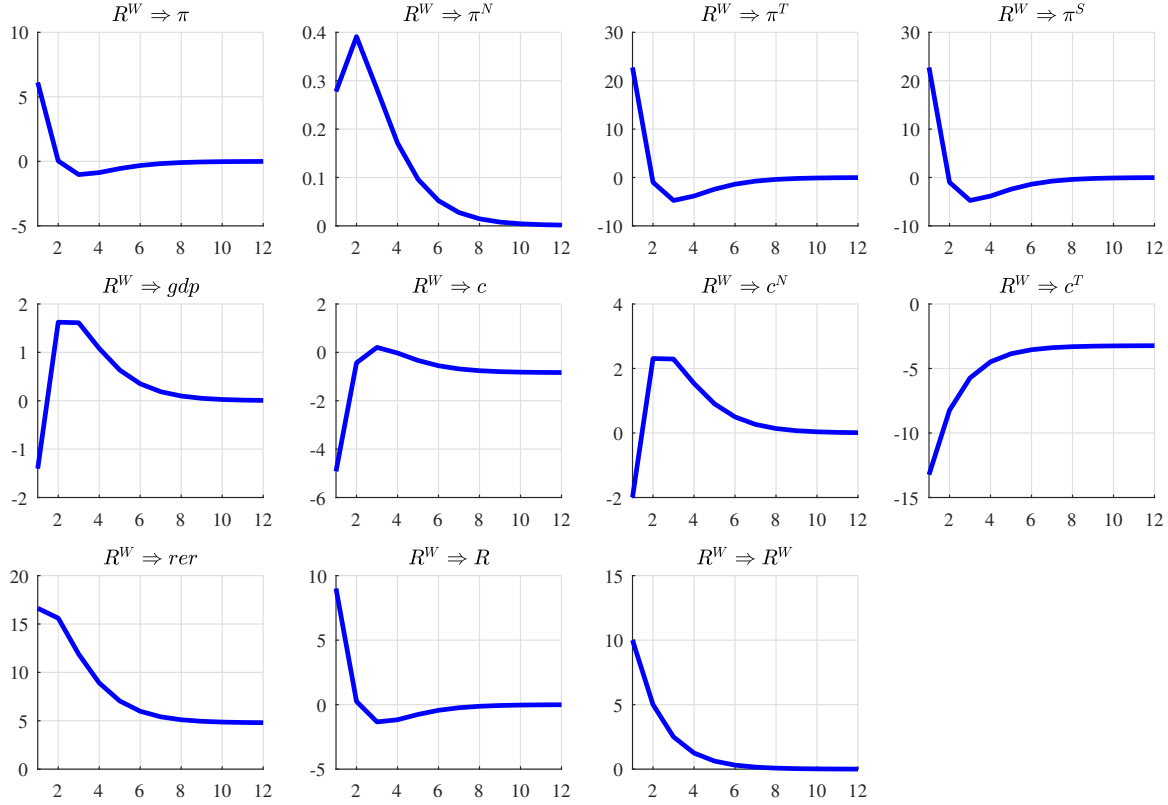
¹⁵Inflation in non-tradables rises due to the policy rule. Under the same calibration, but using a policy rule that targets non-tradable inflation only, it can be shown that non-tradable inflation will not move after the shock, and all the adjustment will come from tradable inflation only.

¹⁶We analyze a negative shock to obtain a nominal depreciation.

¹⁷This will not be the case in the quantitative model, where the domestic price of imports is sticky.

¹⁸Under the chosen parametrization, the consumption of tradables is not affected by a domestic shock due to the assumption that the inter-temporal elasticity of substitution of total consumption is the inverse of the intra-temporal elasticity between tradable and non-tradable goods. It can be shown that under this assumption the consumption of tradables can only be affected by foreign shocks in this model.

Figure 1: IRF to the External Interest Rate



Note: Each graph displays the percentage change, relative to steady state, originated by the shock, in the following variables: total, non-tradable and tradable inflation (π , π^N and π^T), nominal depreciation (π^S), output (gdp), total, non-tradable and tradable consumption (c , c^N and c^T), the (CPI-based) real exchange rate (rer), the policy rate (R), and the variable hit by the shock.

interest rate shock equals one for all horizons. In contrast, the ERPT as a response to foreign inflation is around 0.6 in the first period and decreases over time. This is in line with the distinction we made when analyzing the responses to a shock in foreign inflation.

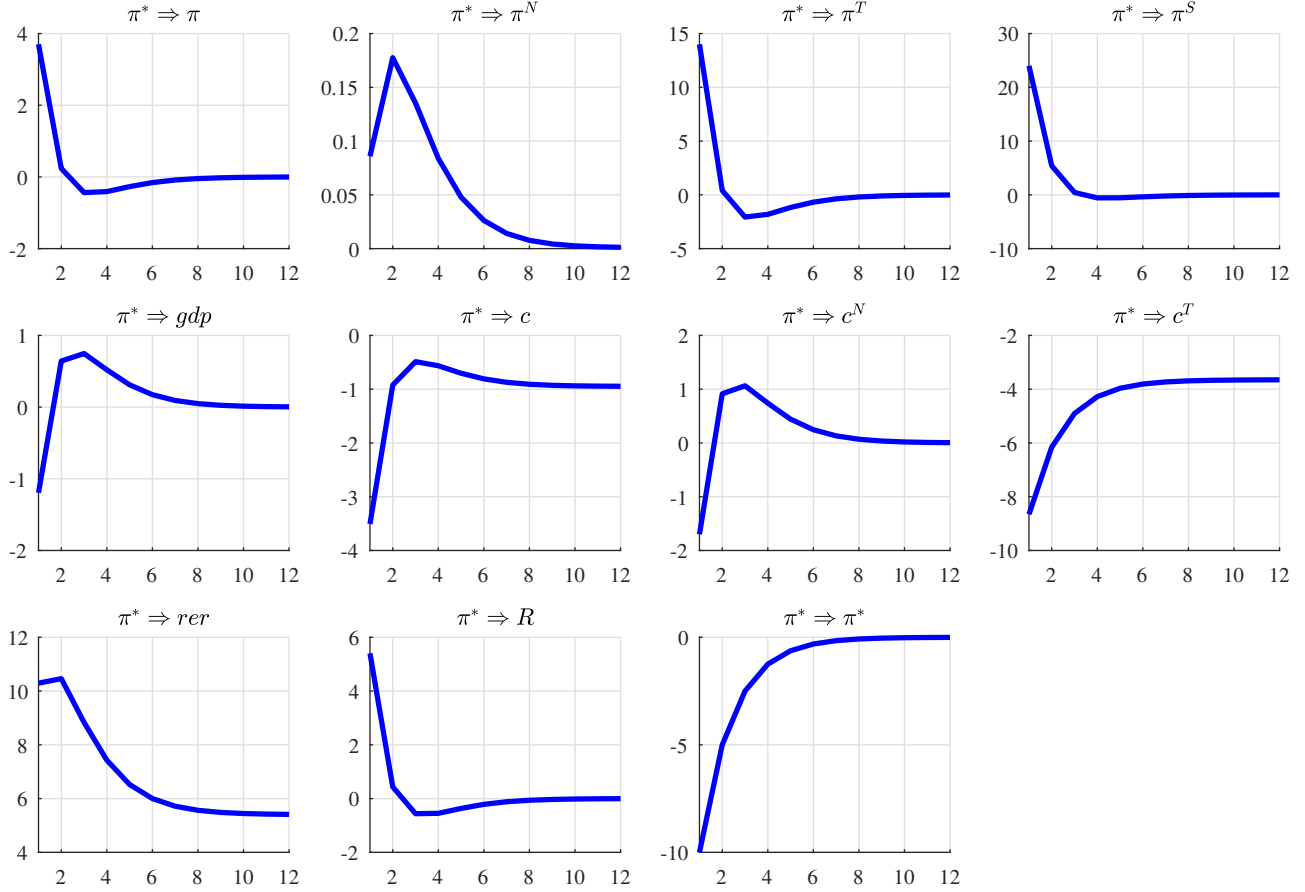
For non-tradable prices, it is also true that the conditional ERPTs in response to a shock in the foreign interest rate and monetary shock are higher than after a foreign-inflation shock; but they are not equal to one. As seen in the figure, it is only for the monetary shock that the ERPT becomes close to one around the 8th quarter, being much lower for the foreign interest rate. Note that as a response to foreign inflation, the ERPT is only 0.02 even after 12 quarters.

Since the CPI is an average of tradable and non-tradable price indices, its conditional ERPT lies between the conditional ERPTs of these two prices. So, for consumer prices, we can see that the highest ERPT is in response to the monetary shock, then to foreign interest rate and then to foreign inflation. Also note that it is increasing in the case of the monetary shock and foreign interest rate, but decreasing in the case of foreign inflation.

In figure 5 we can see the unconditional ERPTs for each price index calculated using the two measures explained in the previous section.¹⁹ As can be inferred from comparing the unconditional ERPTs, in figure 5, with the conditional ones, in figure 4, the shock to foreign inflation explains a higher fraction of the changes in the nominal depreciation rate, and so it has a larger weight in the

¹⁹For the population-VAR measure ($UERPT^{PV}$) the variables included are $\{\pi_t^S, \pi_t, \pi_t^T, \pi_t^N\}$ and the VAR included 15 lags. This number was chosen so that both unconditional measures were similar.

Figure 2: IRF to External Inflation



Note: See Figure 1.

unconditional ERPT measures. This can be appreciated by noticing that the unconditional ERPTs of each price are closer to the ones of that shock than to those of the other shocks.

As discussed in the introduction of the paper, we can see how much information is lost when using the unconditional ERPT measures to predict the effect in prices after a given shock. Only in the case that “the given shock” is a specific combination of the three shocks of the model, the predicted movement in prices using the unconditional ERPTs will be correct. In all other cases, it will be incorrect. How relevant is this bias will depend on which price is being predicted and which shock or shocks hit the economy. In this simple model, it seems that the mistakes using the unconditional measures are less of a problem for tradables in the first quarters, since all the conditional ERPTs are relatively high. This is in part due to the assumption of complete pass-through to domestic tradable prices. In contrast it is more misleading for non-tradables and consumer prices, particularly after a policy shock and at long horizons. In that specific example one would use an ERPT of around 0.05 and 0.16 for non-tradables and consumer prices respectively and the actual values are around 0.9 and 0.95. Overall, even in this simple model, the differences between conditional and unconditional ERPT measures cannot be taken for granted.

Figure 3: IRF to a Monetary Shock

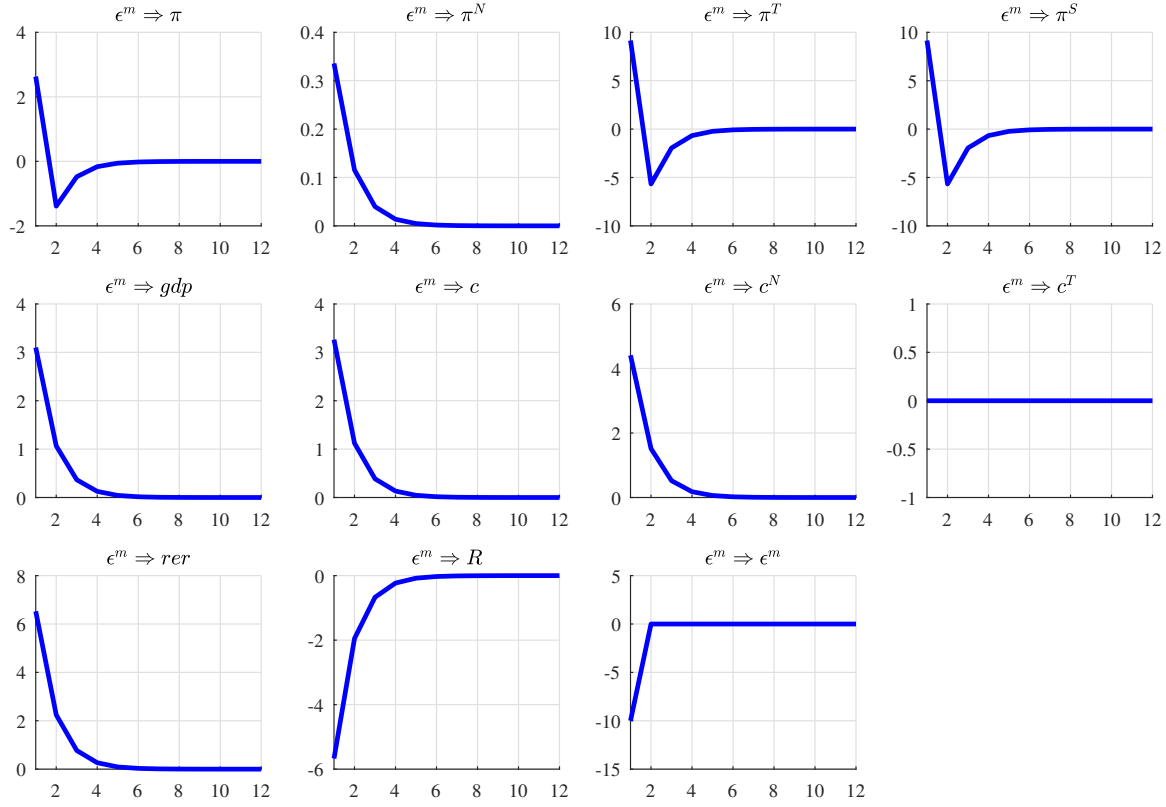
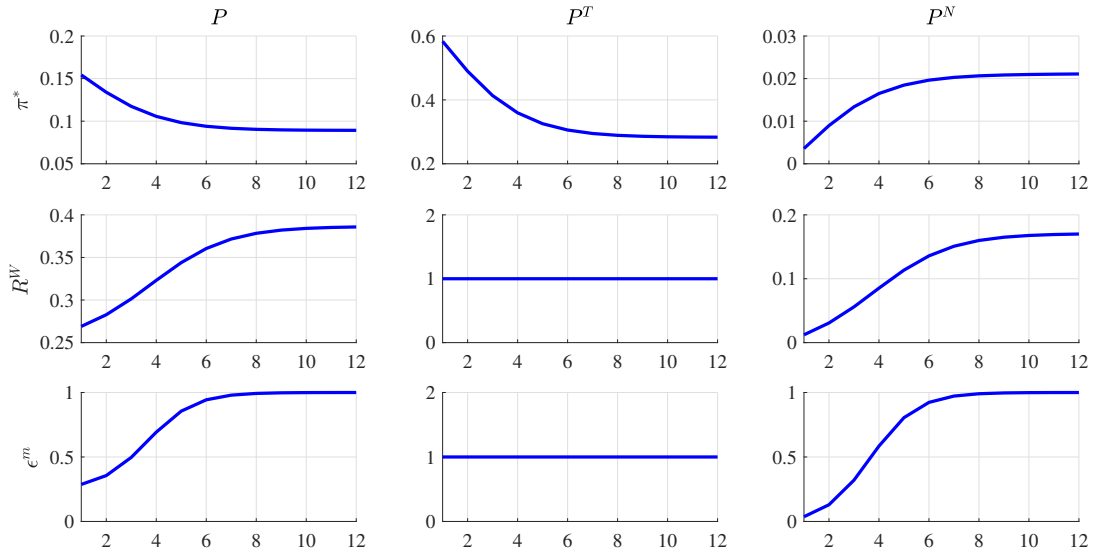
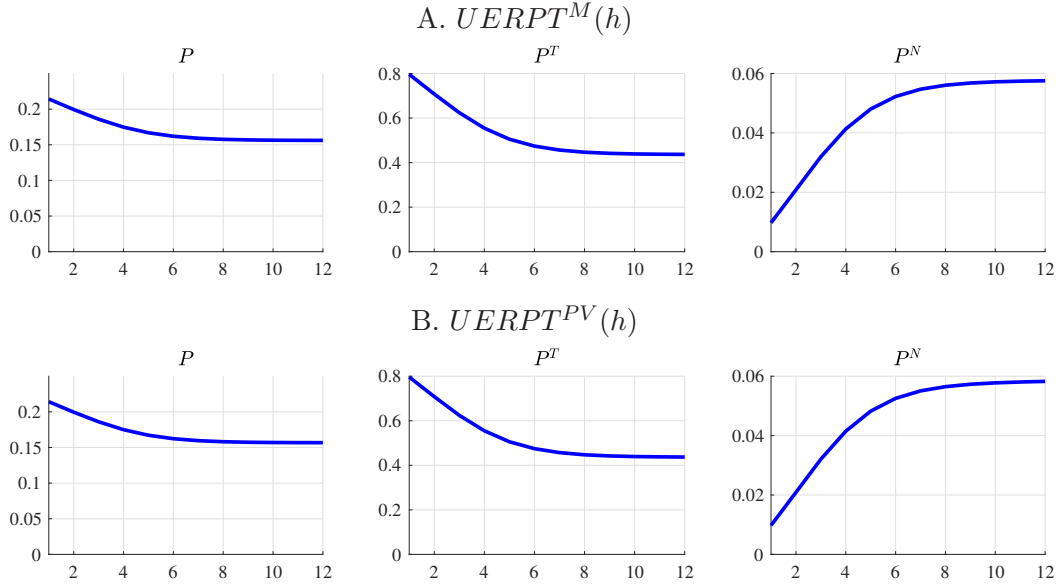


Figure 4: Conditional ERPT



Note: Each graph show the conditional ERPT for the price in each particular column (respectively, CPI, P , tradables, P^T , and non-tradables, P^N), conditional on the shock in each particular row (respectively, foreign inflation, π^* , world interest rate, R^W , and monetary policy, e^M).

Figure 5: Unconditional ERPT



3.3 Importance of Expected Monetary Policy for ERPTs

This subsection shows the importance of taking into account expected monetary policy when discussing ERPTs²⁰. As a first exercise we change the autocorrelation of the policy shock, implying different policy paths relative to the baseline. The second exercise is closer to a real world alternative: it compares the conditional ERPTs to foreign shocks and the unconditional ERPTs in the baseline model with cases when the policy rate, instead of following the rule, is held fixed for a number of periods, starting at the same time the shock hit the economy.

Figure 6 presents the conditional ERPTs to the monetary policy shock in the baseline calibration, as well as the alternatives in which the policy shock displays an autocorrelation of either 0.5 or 0.9²¹. We can see that the ERPTs for non-tradables and total CPI change significantly with more persistent shocks²². When the autocorrelation increases from 0 to 0.5, the ERPTs of P^N and P are not significantly affected in the very short run, but they decrease systematically starting from around the second quarter. This implies that it converges to 1 slower than in the baseline case. When the autocorrelation is further increased, the short run ERPT increases, and then it also converges slower to 1, making the ERPT smaller than the baseline starting around the 3rd quarter.

The second exercise, shown in Figure 7, compares ERPTs when following alternative policy paths. In the baseline, shown in blue, after each shock the policy rate follows the rule, as assumed in the impulse responses in the previous section. Alternatively, we assume that at the time of the shock, the policy maker credibly announces that the policy rate will be maintained fixed (at its steady-state value) for a given number of periods, returning to the Taylor rule afterwards.²³ In the figure, the baseline is contrasted with alternatives in which the interest rate is fixed for 2 and 4 periods. A priori, the effects on ERPTs are not evident. On one hand, fixing the rate following a nominal depreciation

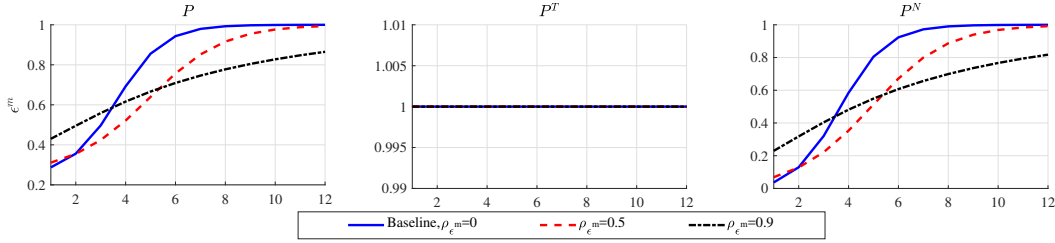
²⁰For IRFs with the alternative policies described in this subsection, refer to Appendix C.

²¹For the models that change the autocorrelation of the monetary shock, the only conditional ERPT that is affected is after a monetary shock.

²²There is no change in the ERPT of the tradable good, since it is one by construction.

²³Computationally, this is implemented by a backward-looking solution as in Kulish and Pagan (2016) or the appendix in Garcia-Cicco (2011).

Figure 6: Conditional ERPT under more persistent policy shocks



Notes: Each graph shows the conditional ERPTs to the monetary shock calculated for models with different values of the autocorrelation of the monetary shock. The blue solid line shows the baseline model with *iid* monetary shocks, the dashed red line shows the model with an autoregressive coefficient of 0.5 and the dash-dotted black line shows the case with an autoregressive coefficient of 0.9.

is more dovish so inflation will likely be higher. On the other, a more dovish policy path induces a higher NER. Therefore, the effect on the ratio computed in the ERPT is unclear.

Figure 7 shows that the effects of alternative policy paths are not monotone. When the interest rate is fixed for 2 periods, the conditional ERPTs are generally higher than when the interest rate follows the Taylor rule. In contrast, when the interest rate is fixed for 4 periods, conditional ERPTs are not only lower than when the interest rate is fixed for 2 periods, but also than the baseline. Moreover, the influence of alternative policy paths seems to affect more the conditional ERPTs after a foreign interest rate shock than after a shock to foreign inflation. As expected, the changes in unconditional ERPTs go in the same direction as the changes in conditional ERPTs.

Overall, we have shown that alternative policy paths can greatly influence ERPTs, both conditional and unconditionally. Thus, it would be much more informative for policy makers if they are presented with alternative ERPT measures, for different choices of future policy paths. The methodologies from the empirical literature cannot produce such an exercise. And while a DSGE model can be used to this end, as we mentioned in the introduction, there is no such analysis available yet in the model-based literature.

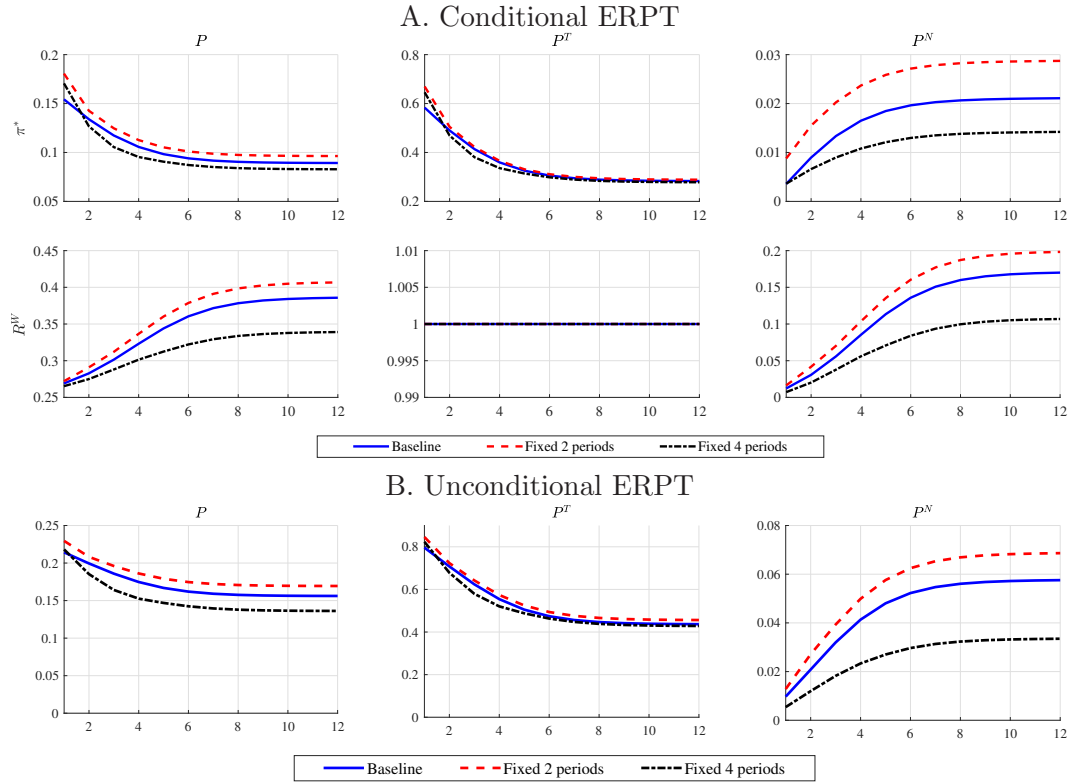
3.4 Sensibility of ERPTs to different parameters

ERPTs, as any other statistic, depend on the dynamics of the model and can crucially change with alternative parameter values. One of the parameters relevant for inflation dynamics in general and for ERPT in particular is indexation to past inflation. The baseline version of the model assumes that the N sector, which is the only sector where prices are set locally, is indexed to the inflation target when prices are not chosen optimally. Instead, we show here how the results change when the non-tradable sector indexes to their own inflation, π_{t-1}^N or to total inflation, π_{t-1} ²⁴.

When indexation is only to the target, the connection between non-tradable prices and the nominal exchange rate is only through a general equilibrium channel. For a given shock, the N market has to clear, and so prices move. If we add indexation to the own inflation when prices are not set optimally, there will be an amplification mechanism at work for the same general equilibrium effect. This is because, after a given shock, for the same change in the nominal exchange rate, the change in non-tradable inflation will be amplified due to indexation. This can be seen in the dashed-black lines in figure 8. Compared to the baseline case, this model shows higher ERPTs in general, with the same general evolution for foreign shocks and an overreaction for the monetary shock.

²⁴For IRFs with alternative indexation dynamics described in this subsection, see Appendix C.

Figure 7: Conditional and Unconditional ERPT fixing the policy rate for T periods



Notes: the graphs show conditional ERPTs to foreign inflation and the foreign interest rate, as well as the unconditional measure $UERPT^M(h)$, for alternative paths of the policy rate. The solid blue line is the baseline model, the dashed red line is the case when the rate is held fixed for two periods, and the dash-dotted black line is the case where it is fixed for 4 periods.

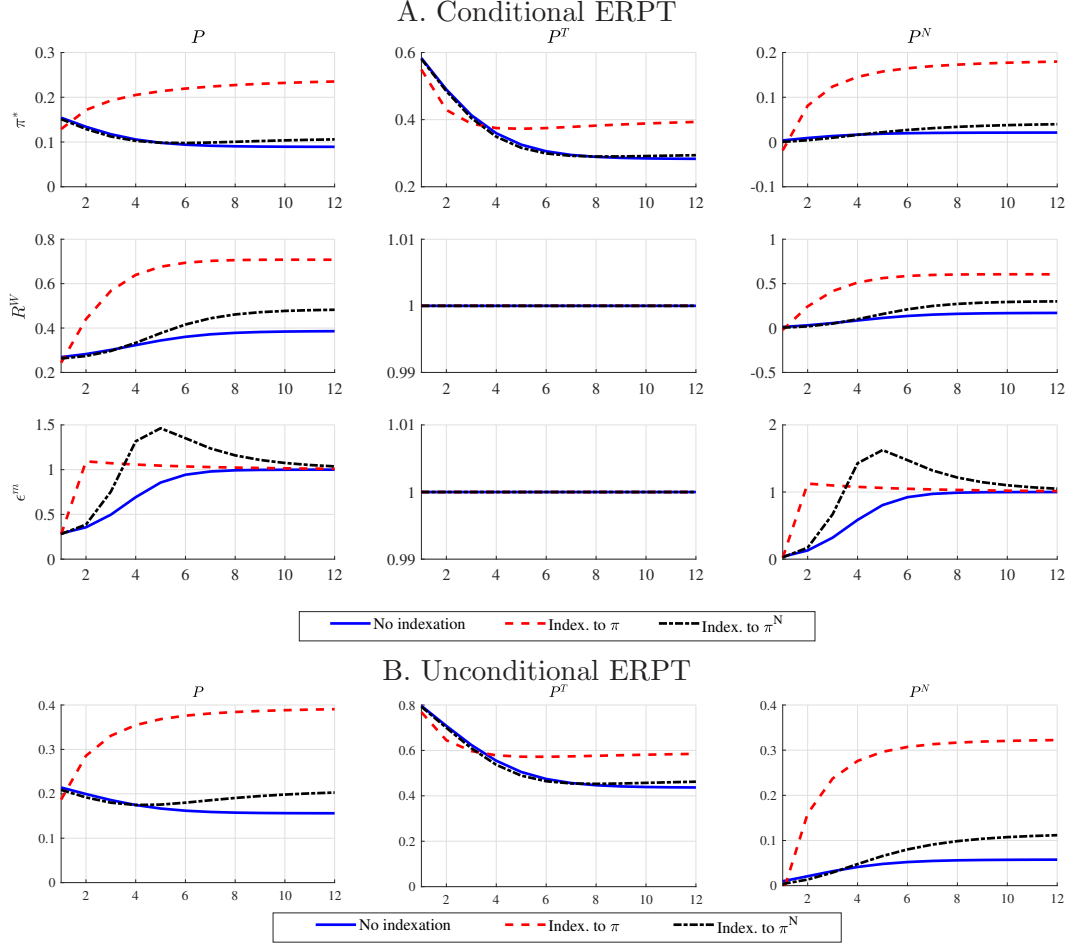
When firms in the N sector are indexed to total inflation there is a significant change in price dynamics, making non-tradable inflation follow with a lag the changes in total inflation. Because of this, in addition to the general equilibrium effect, changes in the exchange rate will have a direct impact on non-tradable inflation, since the indexation of the N sector is now directly affected by the depreciation of the NER. As the ERPTs of tradable prices are generally very high, this change in the model brings a significant increase in the ERPTs of non-tradable prices as well as for CPI. This is true for both conditional and unconditional ERPTs, and particularly important for the ERPT conditional on foreign shocks.

There are other model features that can have a direct impact on ERPTs. Some of these are introduced in the quantitative model of the next section, such as using imported inputs in the production of local goods, introducing price rigidities in the imported sector, using importable goods in investment, nominal rigidities and indexation in wages, among others.

4 The Quantitative DSGE Model

As we have argued, the shortcomings of the empirical approach to ERPTs are of quantitative nature, and therefore we need a model that matches satisfactorily the dynamics observed in the data. To that end, in this section we reproduce the analysis presented with the simple model using a DSGE model estimated for Chile. Given that the model is relatively large, here we present an overview

Figure 8: Conditional and Unconditional for Alternative Parameters Concerning Indexation



Notes: the graphs show conditional ERPTs, as well as the unconditional measure $UERPT^M(h)$, for models with different indexation dynamics. The blue solid line shows the baseline model, which has indexation to the inflation target, the dashed red line shows the model with indexation to total inflation and the dash-dotted black line shows the case with indexation to sectoral N inflation.

of the model, leaving to the Appendix D the full description, as well as the equilibrium conditions, the parametrization strategy and goodness-of-fit analysis. We then proceed by analyzing what are the main driving forces behind exchange rate fluctuations in the model, and provide intuition on how these shocks propagate to the economy. The comparison between conditional and unconditional ERPTs is performed next, and we finish by analyzing how alternative policy paths influence ERPTs.

4.1 Model Overview

Our setup is one of a small open economy with both nominal and real rigidities, and incomplete international financial markets. There are three goods produced domestically: commodities (Co), non-tradables (N), and an exportable good (X). The first is assumed to be an exogenous endowment that is fully exported, while the other two are produced by combining labor, capital, imported goods (M , which are sold domestically through import agents) and energy (E). Consumption (both private and public) and investment goods are a combination of N , X and M goods.²⁵ The model features

²⁵Final consumption also requires Energy and Food, which are the items that are considered in the non-core part of inflation in Chile. These are assumed to be produced by combining X and M goods; although having a different price

exogenous long run-growth under a balanced growth path, although we allow for sector-specific trends in the short-run.

Households derive utility from consumption and leisure, borrow in both domestic and foreign-currency-denominated bonds, and have monopoly power in supplying labor. Moreover, we assume imperfect labor mobility across sectors. Household's utility exhibits habits in consumption, and investment is subject to convex adjustment costs.

Firms in the X , N and M sectors are assumed to have price setting power through a monopolistic-competition setup. The problem of choosing prices, as well as that of setting wages, is subject to Calvo-style frictions, with indexation to past inflation. As discussed above, the possibility of indexation to aggregate inflation is relevant to determine the ERPT to different goods, particularly non-tradables. Accordingly, we allow indexation to both past CPI and own-sector inflation, as well as the target, estimating the parameters that govern the relative importance of each of these indexations.

Monetary policy sets the interest rate on domestic bonds, following a Taylor-type rule that responds to past policy rate (smoothing), deviations of CPI and core inflation from the target, and the growth rate of GDP relative to its long-run trend. Fiscal policy is assumed to finance an exogenous stream of consumption using lump-sum taxes and proceedings from the ownership of part of the commodity production. The final relevant agent is the rest of the world, where international prices and interest rates are set exogenously, following the small-open economy assumption.

The model features 24 shocks, both of domestic and foreign origin. These are:

- Domestic (15): consumption preferences, labor supply (X and N), stationary productivity (X y N), long run trend, desired markups (M , X and N), endowment of commodities, relative prices of food and energy, efficiency of investment, government consumption, and monetary policy.
- Foreign (9): world interest rate (risk free), foreign premium (2 shocks, described later), international prices of commodities, imported goods and CPI for trade partners (4 shocks, described later), demand for exports of X , GDP of trade partners.

All these variables are assumed to be AR(1) processes, with the exception of international prices which we describe below.

The parameter values are chosen by a combination of calibration and Bayesian estimation. We use data for Chile, at a quarterly frequency from 2001.Q3 to 2016.Q3. The data includes aggregate variables for activity, inflation, interest rates and the exchange rate, as well as sectoral series for activity, prices and wages. The dataset also includes international variables such as interest rates, prices and GDP of trading partners. In the appendix we include a complete description of the model and the parametrization strategy. Moreover, we also show that the estimated model can satisfactorily match second moments for the relevant observables in the data.²⁶

4.2 Main Drivers of the NER and Implied Dynamics

As we discussed before, the analysis of the ERPT requires to first identify the main shocks driving the movements in the NER. While the model features a large number of shocks, the estimation indicates that five shocks can explain almost 95% of the variance of the nominal depreciation. Of these five, four are related with the uncovered interest rate parity in the model (which we later describe): the world interest rate (R^W), two types of risk premia (country premium, $C.P.$, and deviations from UIP), and monetary policy ($M.P.$). The other is a common trend in international prices denominated in dollars (ΔF^*), which we describe in more detail below. In what follows, we first show the relative importance

dynamic in the short run.

²⁶All the results presented in the following subsections use the posterior mode as the parameter values.

of each of these by means of a variance-decomposition exercise, and then provide intuition for their propagation mechanism.

Table 2 shows the contribution of these five shocks to account for the unconditional variance of the NER depreciation (π^S). In addition, we show the contribution of these shocks in the variance decomposition for alternative inflation measures, the policy rate and the real exchange rate.

Table 2: Variance Decomposition

Var.	$M.P.$	R^W	$C.P.$	UIP	ΔF^*	Sum.
π^S	3	8	2	13	67	94
π	3	12	3	5	8	31
π^T	4	19	5	9	14	50
π^M	3	17	5	8	13	46
π^N	2	13	3	2	6	27
R	18	18	5	5	10	56
rer	3	15	4	11	15	48

Note: Each entry shows the % of the unconditional variance of the variable in each row, explained by the shock in each column, computed at the posterior mode. The shocks correspond to monetary policy ($M.P.$), world interest rate (R^W), country premium ($C.P.$), deviations from UIP (UIP) and the trend in international prices (ΔF^*). The variables are: nominal depreciation (π^S), total, tradable, imported and non-tradable inflation (respectively, π , π^T , π^M and π^N), the policy rate (R) and the real exchange rate (rer).

As can be seen, the shock that contributes more to NER fluctuations is the trend in international prices (ΔF^*), explaining almost 70% of its variance. The risk shock that emerges as deviations from the interest parity (UIP), as well as the world interest rate (R^W), also explain a non trivial part of the volatility of π^S . Together the three account for almost 90% of the variance of the NER. These shocks also play a non trivial role in accounting for inflation variability, explaining around 50% of tradable inflation, almost 30% of non-tradable, and 30% of total CPI, as well as a non-trivial fraction of the variance of R and rer . Thus, while clearly not the only relevant factors, the determinants of the NER important for inflation fluctuations as well.

A relevant distinction is that, while the shock to the trend in international prices is the most relevant for the NER, its relative contribution for inflation is smaller. This is because the flexible exchange rate acts as a buffer to nominal external shocks, isolating, to a large extent, domestic variables from their influence. This distinction will be crucial for the conditional vs. unconditional ERPT analysis below.

Next, we discuss how these shocks enter in the model, and the dynamics they generate. The model features three international prices denominated in dollars: commodities (P_t^{Co*}), imported goods (P_t^{M*}), and CPI of commercial partners (P_t^*).²⁷ These prices need to cointegrate because relative prices are stationary in the model. Specifically, we assume the following structure for these prices:²⁸

$$\hat{P}_t^j = \Gamma_j \hat{P}_{t-1}^j + (1 - \Gamma_j) \hat{F}_t^* + u_t^j, \quad (8)$$

²⁷This last price is the relevant reference price for exports of X goods, and it also the external price used for the definition of the real exchange rate, $rer_t = \frac{S_t P_t^*}{P_t}$.

²⁸A hat denotes log-deviations relative to its long-run trend.

$$\begin{aligned}\Delta \hat{F}_t^* &= \rho_{F^*} \Delta \hat{F}_{t-1}^* + \epsilon_t^{F^*}, \\ u_t^j &= \rho_j u_{t-1}^j + \epsilon_t^j,\end{aligned}$$

for $j = \{Co*, M*, *\}$ and ϵ_t^j are *i.i.d.* exogenous shocks. Under this specification, each price is driven by two factors: a common trend (F_t^*) and a price-specific shock (u_t^j). The parameter Γ_j determines how slowly changes in the trend affect each price. The presence of a common trend generates co-integration among prices (as long as $\Gamma_j < 1$), and the fact that the coefficients (8) add-up to one forces relative prices to remain constant in the long run²⁹. While in principle both the trend and the price-specific shocks can affect all variables in the model, according to the estimation, only the trend is quantitatively relevant to explain fluctuations in the NER.

This specification for international prices is more complex than in the simple model of the previous section, however ΔF^* qualitatively resembles the shock to inflation of traded goods (π^*). Thus, the intuition behind the effect of shocks to ΔF^* is similar to that of π^* in the simple model. Figure 9 shows impulse responses to a shock of ΔF^* . After a negative shock to the international trend in prices, aggregate demand falls. As the market for non-tradable goods has to clear domestically, the shock generates a fall in the relative price of non-tradables, a real exchange rate depreciation, a drop in the production of N , an increase in the output of X , and an overall fall in GDP. Moreover, given the real depreciation and the presence of price rigidities, the nominal exchange rate depreciates as well.

To explain the dynamics of inflation first note that, without indexation, the required fall in the relative price of non-tradables would lead to an increase in the price of tradables (due to the nominal depreciation) and a drop in the price of non-tradables, which can actually be observed in the very short run. But with indexation to aggregate inflation (in both wages and prices), inflation of non-tradables starts to rise after a few periods.³⁰ Therefore, the indexation channel affects significantly the dynamics of inflation (and the ERPT) in the non-tradable sector. Finally, given the monetary policy rule, the domestic interest rate increases to smooth the increase in inflation.

The other shocks are associated with the uncovered interest rate parity, which up to first order can be written as,³¹

$$\hat{R}_t = \hat{R}_t^W + E_t \{ \hat{\pi}_{t+1}^S \} + \phi_b \hat{d}_t^* + \hat{\xi}_t^{R1} + \hat{\xi}_t^{R2}.$$

Here \hat{R}_t is the domestic rate, \hat{R}_t^W is the foreign risk free interest rate, $E_t \{ \hat{\pi}_{t+1}^S \}$ is the expected nominal depreciation, and $\phi_b \hat{d}_t^*$ is a premium elastic to foreign debt, \hat{d}_t^* , which acts as the closing device. Additionally, there are two risk premium shocks $\hat{\xi}_t^{R1}$ and $\hat{\xi}_t^{R2}$. They differ in that the first one is matched with a measure of the country premium in the data (the JP Morgan EMBI Index for Chile),³² while the second is unobservable and accounts for all other sources of risk that explain deviations from the EMBI-adjusted interest rate parity. In the tables and figures $\hat{\xi}_t^{R1}$ is labeled as *C.P.* and $\hat{\xi}_t^{R2}$ is called *UIP*.

Figure 10 shows the responses to a positive realization of the *UIP* shock, which is qualitatively analogous to the influence of a shock to the world-interest-rate in the simple model³³. This shock increases the cost of foreign borrowing, which triggers both income and substitution effects, leading to a contraction in aggregate demand. This leads to both real and nominal depreciations, and a reduction

²⁹The usual assumption for these prices in DSGE models with nominal rigidities is obtained as a restricted version of this setup explained in Appendix D.

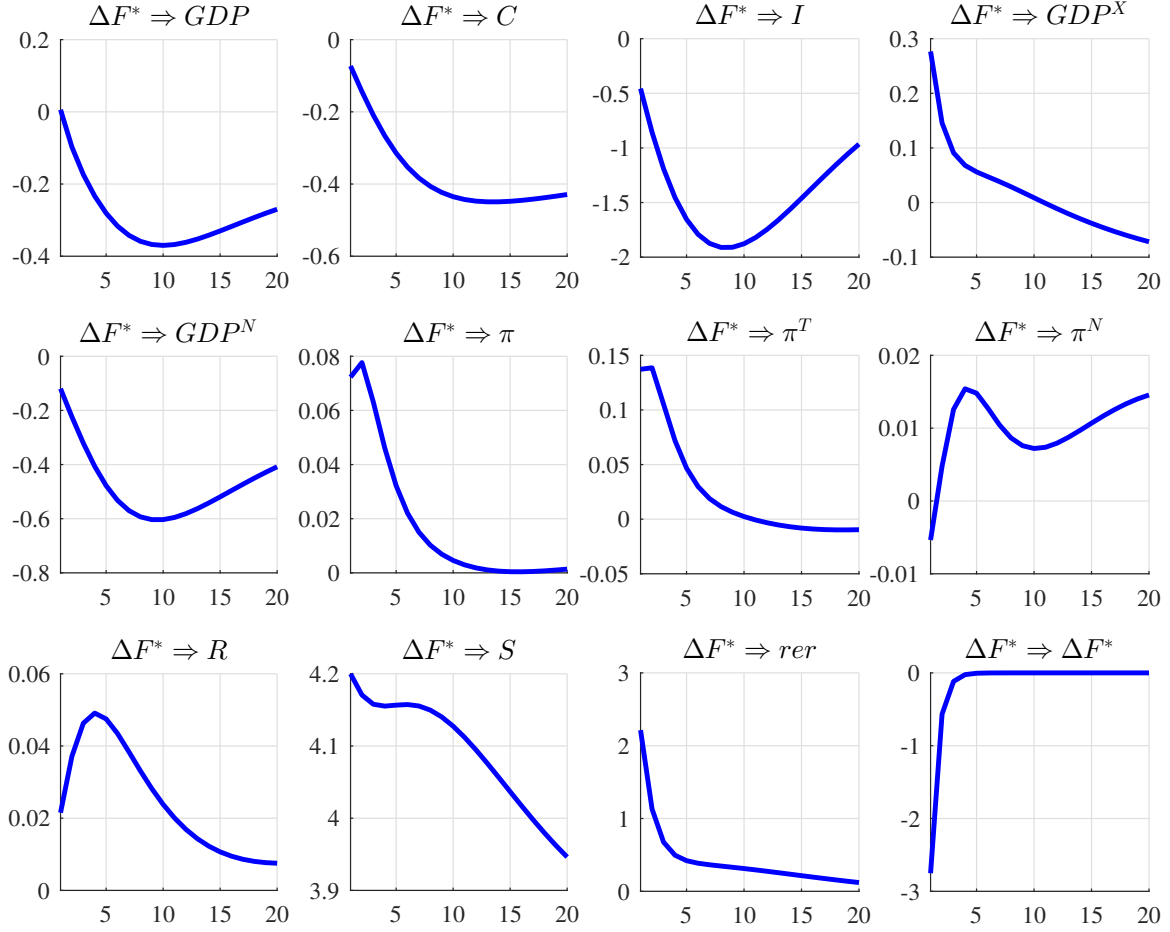
³⁰The fraction of prices and wages in the N sector that are indexed to aggregate inflation per period is around 18% and 11% respectively. One can numerically show that if these were set to zero, the response of π^N would be negative for the relevant horizon.

³¹A hat denotes log-deviations relative to steady state.

³²Specifically, the EMBI index is matched with $\phi_b \hat{d}_t^* + \hat{\xi}_t^{R1}$.

³³The responses to shocks R^W and *C.P.* in the quantitative model are similar to those originated by a *UIP* shock, and thus are omitted to save space.

Figure 9: IRF to a drop in the trend of international prices

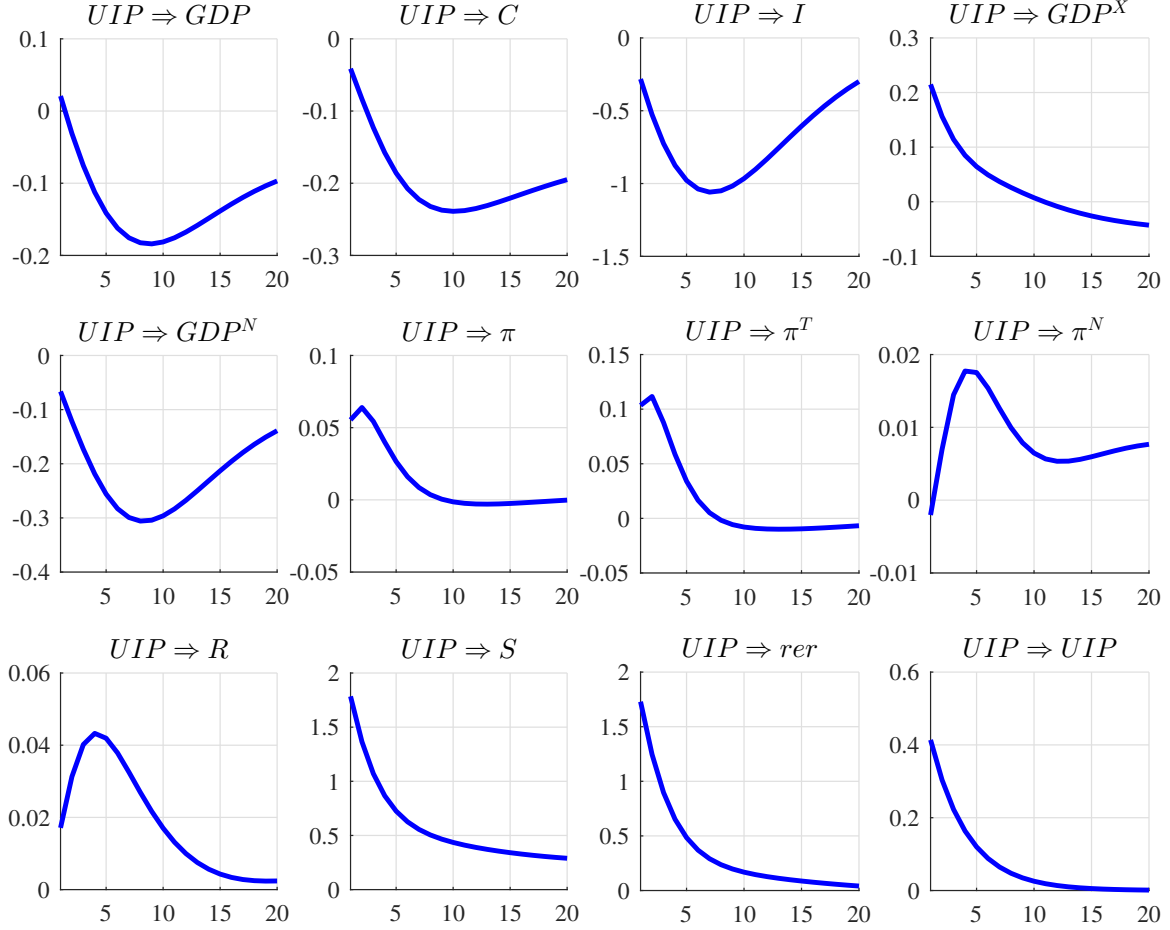


Note: Each graph presents the impulse response function, computed at the posterior mode, expressed as percentage deviations relative to the steady-state. The variables are GDP, consumption, investment, GDP in the X and the N sectors, total inflation, tradables and non-tradables inflation (excluding food and energy), the monetary policy rate, and the nominal exchange rate, the real exchange rate, and the variable being shocked. The size of the shock is equal to one standard-deviation.

in all measures of activity; except for production in X that is favored by the reallocation of resources from the N sector. All measures of inflation increase, and the role of indexation in explaining π^N is similar to what we described before. Accordingly, the policy rate rises after this shock.

We conclude by reminding that, as discussed before, even though both shocks have an impact through aggregate demand, the shock to ΔF^* has also a direct impact on inflation that dampens the effect generated by NER changes. In this more complex model, this happens through two different channels. First, a drop in international prices puts downward pressure to the domestic price of imports. Second, given the presence of imported inputs in the production of both X and N , a reduction in world prices will, *ceteris paribus*, reduce the marginal cost in these sectors, dampening also the response of X and N inflation. Thus, as in the simple model, shocks to international prices are expected to have lower conditional ERPTs that shocks to the interest rate parity condition.

Figure 10: IRF to a positive risk shock (deviations from UIP)



Note: See Figure 9.

4.3 Conditional vs. Unconditional ERPTs

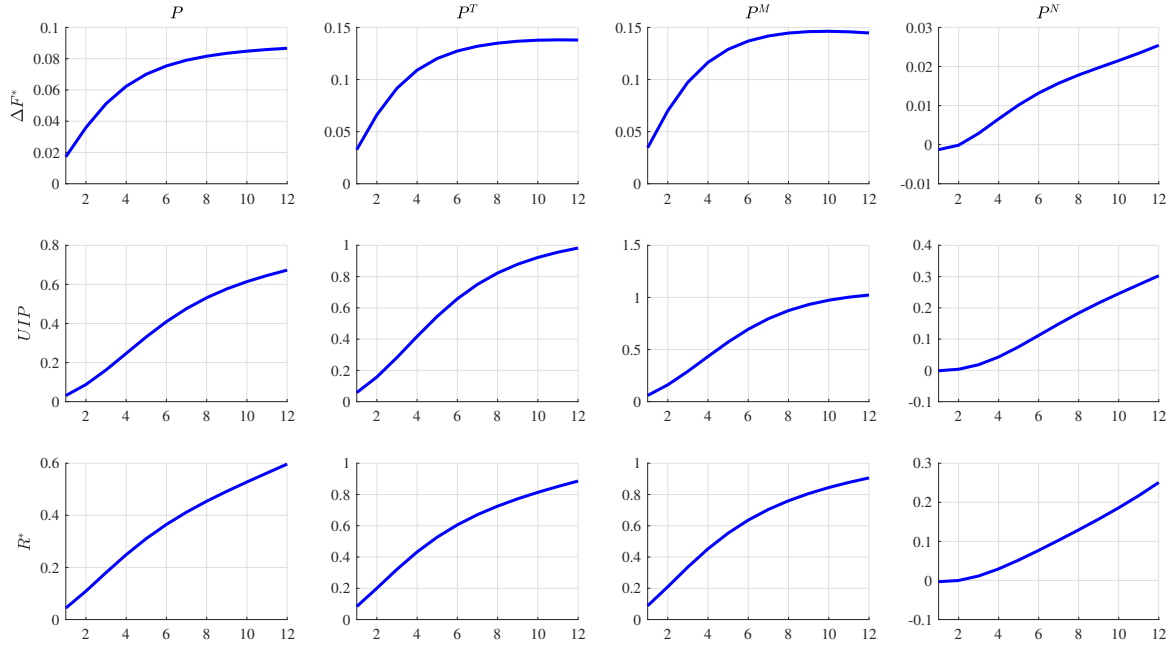
We begin by computing the conditional ERPTs associated with the three main shocks behind fluctuations in the NER. We present the results for aggregate CPI (P), tradables (P^T), imported (P^M) and non-tradables (P^N), the last three excluding Food and Energy. In line with the previous discussion, and as can be seen in figure 11, the conditional ERPTs generated by ΔF^* are significantly different from those implied by shocks to the UIP and to the world interest rate R^W . For a horizon of 2 years, the conditional ERPT given a shock to international prices is less than 0.1 for total CPI, smaller than 0.05 for non-tradables, and close to 0.15 for both traded and imported goods.

In sharp contrast, for the same horizon, the conditional ERPTs to the UIP shocks are much higher for all prices: close to 0.5 for CPI, larger than 0.8 for tradables and importables, and near 0.2 for non-tradables. For the world-interest-rate shock the conditional ERPTs are somehow smaller, but still larger than those obtained after a shock in the trend of international prices.

Figure 12 displays both measures of unconditional ERPTs we introduced in Section 2: panel A shows the weighted average of conditional ERPTs, while panel B displays the measure obtained using the Population VAR approach.³⁴ In line with our previous analysis, both measures of unconditional

³⁴The VAR is assumed to contain the following variables: world interest rate (R^W), foreign inflation (π^*), inflation of commodities (π^{Co*}) and imports (π^{M*}), growth of external GDP (Y^*), nominal depreciation rate (π^S), and inflations

Figure 11: Conditional ERPT



ERPT lie between the conditional measures reported before.³⁵ Moreover, the empirical VAR literature using Chilean data estimates an ERPT close to 0.2 for total CPI after two years, with a similar value for tradables and close to 0.05 for non-tradables.³⁶ These are close to the measures of unconditional ERPTs we report here.

Overall, the evidence presented in this section confirms the intuition developed with the simple model: conditional ERPTs are quite different from those obtained from aggregate ERPT measures comparable to those in the literature. Thus, using the results from the empirical literature will almost surely lead to biases in the estimated dynamics of inflation following movements in the NER. In turn, the analysis can be greatly improved by an assessment of which shocks are behind the particular NER change, and the use of conditional ERPT measures.

4.4 ERPT and Expected Monetary Policy

Our second concern regarding the use of the ERPT obtained from the empirical literature is that it could mistakenly lead to thinking that actual and future monetary policy has little to say about the behavior of both the NER and prices. Conceptually, this discussion is independent from the potential differences between conditional and unconditional ERPTs; although we will see that quantitatively the source of the shock also matters for this discussion.

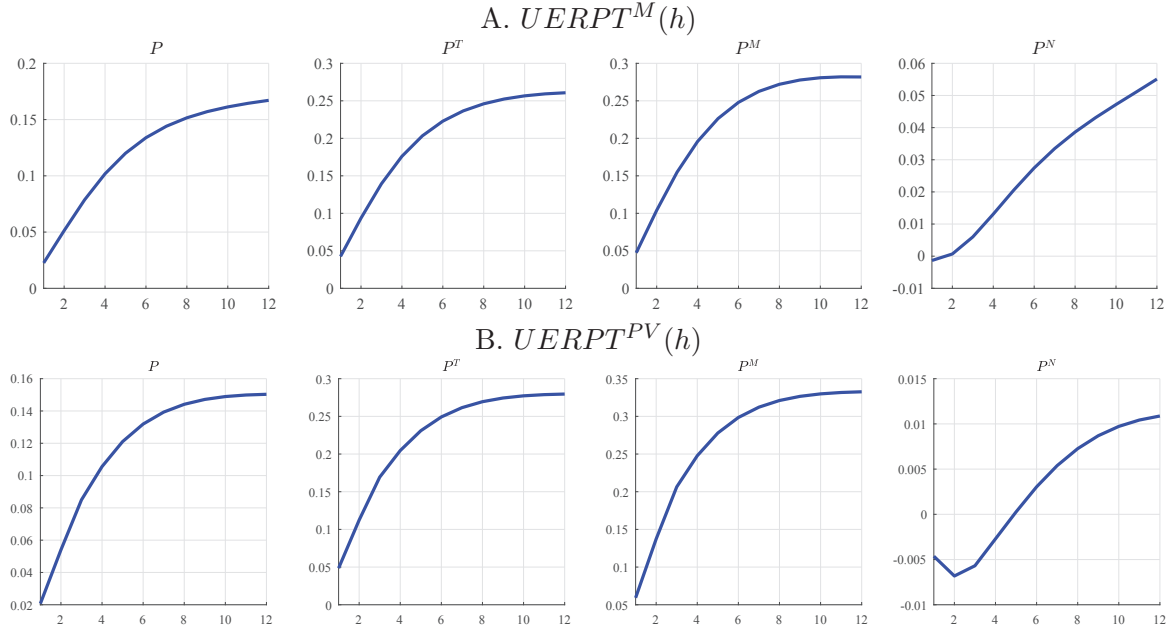
The starting point is to notice that, as discussed in Section 4.2, in the benchmark model the monetary policy rate increases (and it is expected to remain high) in response to the main shocks that depreciate the currency. We compare the benchmark ERPTs, obtained assuming the policy rate

for CPI (π), tradables (π^T), importables (π^M) and non-tradables (π^N). These series are those used in the empirical literature. The ERPT is computed using the shock for π^S in the Cholesky decomposition. We ran a VAR(2) based on the BIC criterion.

³⁵Although the measure $UERPT^M(h)$ includes all shocks, the main drivers of the unconditional measures are ΔF^* , UIP and R^W .

³⁶See, for instance, [Justel and Sansone \(2015\)](#), [Contreras and Pinto \(2016\)](#), [Albagli et al. \(2015\)](#), among others.

Figure 12: Unconditional ERPT



follows the estimated rule, with alternative scenarios that deviate temporarily. In particular, as we did with the simple model, it is assumed that, when the shock hits the economy, the central bank announces that it will maintain the interest rate at its pre-shock level for T periods, and return to the estimated rule afterwards.

Figure 13 shows how selected impulse-response functions change with these policy alternatives, for the main shocks that drive the NER. As in the simple model, the reaction of the ERPTs are not ex-ante evident, since the figure shows that a more dovish policy increases both inflation and the NER.

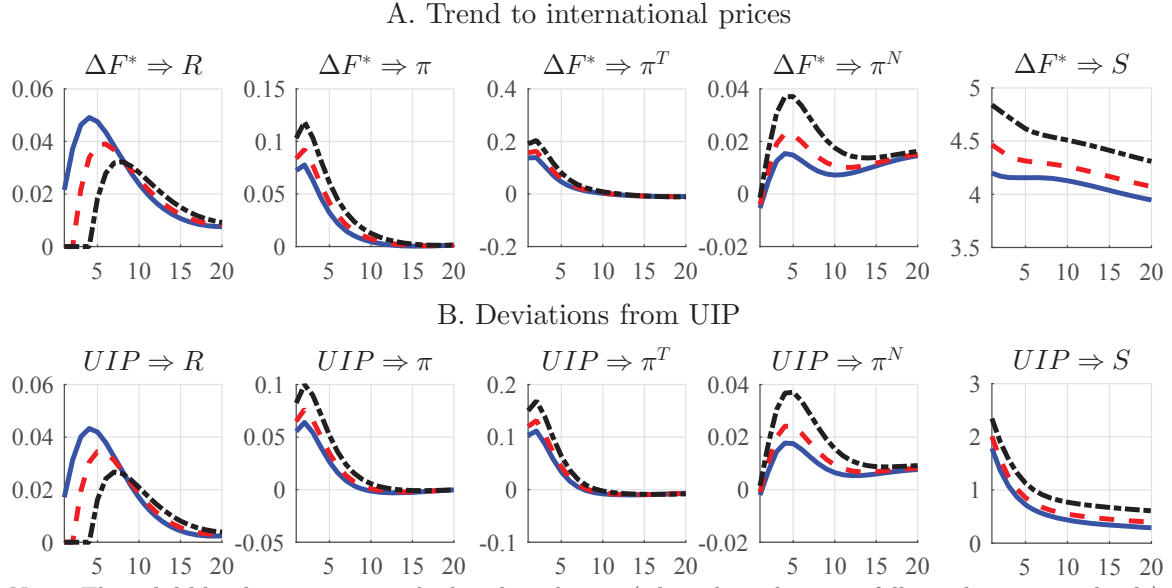
As shown in figure 14, when the shock to the trend in international prices hits the economy, conditional ERPTs vary significantly depending on the reaction of monetary policy. For instance, after two years, the ERPT to total CPI almost doubles if the policy rate remains fixed for a year; and the difference is even larger for non tradables. At the same time, conditional on shocks to either the UIP or the world interest rate, the ERPT measures do not seem to vary significantly as monetary policy changes; except for non-tradables where we can see some differences.

In Figure 15 we compute the unconditional ERPT using the weighted average measure as in (7).³⁷ As can be seen, influenced mainly by the behavior of the ERPT after the shock to international prices, the unconditional ERPT also increases with a more dovish policy. This comparison provides yet another reason to properly account for the source of the shock and to compute conditional ERPTs, as the effect of alternative policy paths will be relevant depending on the shock.

In sum, this analysis highlights that, in thinking about how monetary policy should react to shocks that depreciate the currency, a menu of policy options and their associated conditional ERPT should be analyzed. For some shocks, monetary policy has a significant role to determine the final outcome of both inflation and the NER. As we have argued, this kind of analysis cannot be performed using

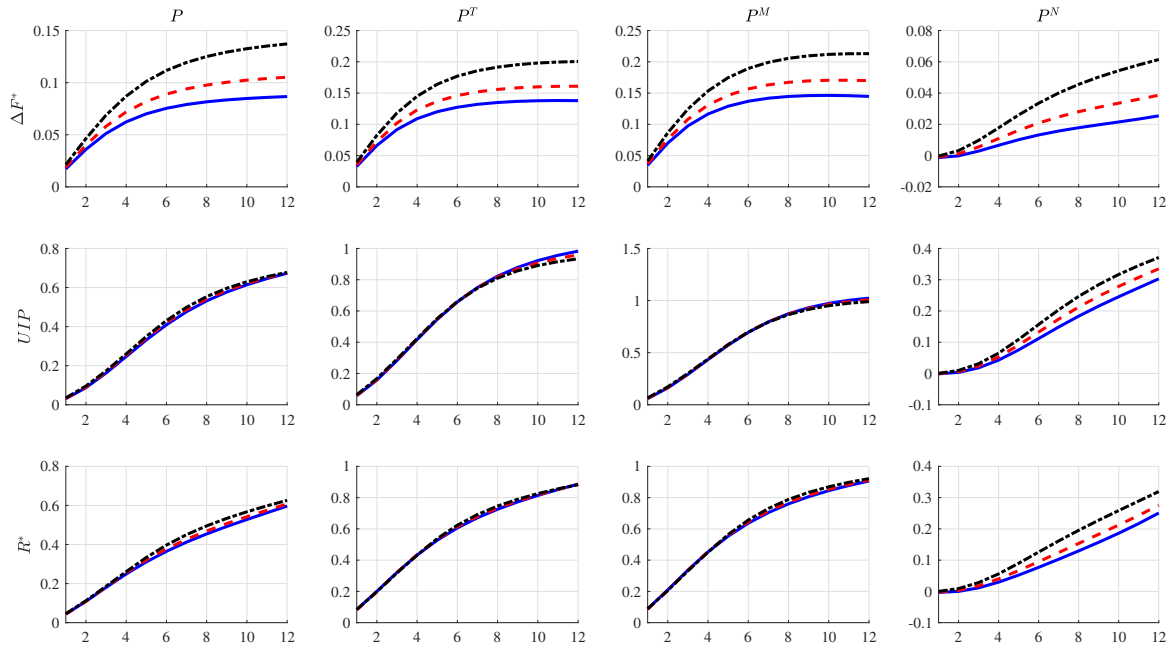
³⁷In this computation, we exclude the monetary policy shock in all models, as it plays no role once we fix the policy rate, and we maintain the weights as in the baseline to isolate the changes only due to different dynamics with alternative policy paths. The Population VAR measure of aggregate ERPT will not vary with this policy comparison, as the alternative paths for the interest rate will only affect the dynamics in the short run, without changing the population moments.

Figure 13: IRF under alternative policy paths



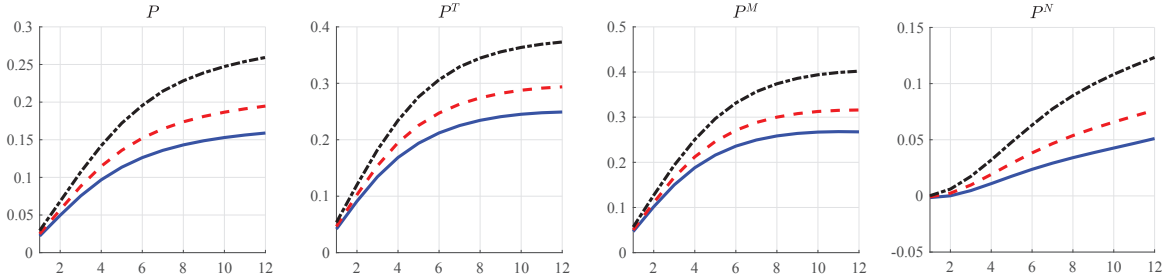
Note: The solid-blue line represents the benchmark case (when the policy rate follows the estimated rule), the dashed-red line is the case in which the rate is fixed for two periods, and the dashed-dotted-black line is when the rate is fixed for 4 periods. The variables shown are the policy rate, total, tradable and non-tradable inflations, and the nominal exchange rate.

Figure 14: Conditional ERPT, under alternative policy paths



Note: See Figure 13

Figure 15: Unconditional ERPT, under alternative policy paths



Note: See Figure 13

the tools and results from the empirical literature, and the related literature using DSGE models has not analyzed the role of alternative policy paths for the ERPT.

5 Conclusions

This paper was motivated by the widespread use of ERPT measures generated by empirical, reduced-form methodologies for monetary-policy analysis. We highlighted two potential problems: the dependence of ERPTs on the shock hitting the economy (separating conditional and unconditional ERPTs), and the influence of alternative expected paths of monetary policy. We first established the relationship between ERPT measures used in the empirical literature with related objects obtained from general equilibrium models. We then used a simple model to conceptually understand how the two shortcomings that we highlight arise in any model. Finally, to assess the quantitative importance of making these distinctions, we used a DSGE model estimated with Chilean data. We found that these distinctions are indeed relevant, and that a policy maker using the results from the empirical literature alone is probably basing her decision on inappropriate tools.

Another way to frame this discussion in a more general context is the following. From the point of view of general equilibrium models, one can define alternative measures of what “optimal” policy means and then fully characterize how monetary policy should respond to particular shocks hitting the economy, in order to achieve the optimality criteria. In that discussion, structural parameters, the role of expectation formation, the nature of alternative driving forces, among other details, will be relevant to determine the path that monetary policy should follow. However, as the empirical measure of the ERPT computed in the literature is, in one way or another, a conditional correlation and not a structural characteristic of the economy, all the relevant aspects of optimal monetary policy can be described without using the concept of ERPT at all. Thus, while the results of the empirical literature can be useful for other discussions in international macroeconomics, its relevance for monetary policy analysis is more limited.

Finally, it is our perception that the role of expected policy to determine the ERPT has not been properly considered in actual policy making. To a large extent, the realized ERPT after a given NER movement can be influenced by monetary policy. However, the widespread use of empirical measures of ERPT for policy analysis, which completely omits this issue, indicates that this is not the way policy makers think about the ERPT. In that way, a fruitful venue for future research could be to study particular episodes of large depreciations, to estimate the extent to which the expected path of policy perceived at the time of the NER movement influenced the dynamics of inflation that followed.

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A ERPT in VARs and DSGE Models

A.1 Conditions for an Exact Relationship

The linearized solution of a DSGE model takes the form

$$c_t = As_{t-1} + Be_t, \quad (\text{A.1})$$

$$s_t = Cs_{t-1} + De_t, \quad (\text{A.2})$$

where s_t is a $n \times 1$ vector of predetermined variables, both endogenous and exogenous, c_t is a $r \times 1$ vector of non-predetermined variables, e_t is a $m \times 1$ vector of *i.i.d.* exogenous shocks (with $E(e_t) = 0$, $E(e_t e_t') = I$, and $E(e_t e_j') = 0$ for $t \neq j$), while A , B , C and D are conformable matrices. The solution in (5) can be obtained by defining

$$y_t = \begin{bmatrix} c_t \\ s_t \end{bmatrix}, \quad F = \begin{bmatrix} 0 & A \\ 0 & C \end{bmatrix}, \quad Q = \begin{bmatrix} B \\ D \end{bmatrix}.$$

Let x_t be a $k \times 1$ vector collecting variables from either s_t or c_t , such that $x_t = S[c_t' s_t']' = Sy_t$ for an appropriate selection matrix S . From (A.1) and (A.2),

$$x_t = \bar{A}s_{t-1} + \bar{B}e_t, \quad (\text{A.3})$$

with

$$\bar{A} = S \begin{bmatrix} A \\ C \end{bmatrix}, \quad \bar{B} = S \begin{bmatrix} B \\ D \end{bmatrix}.$$

If $k = m$ (i.e. the same number of variables in x than shocks in the model), under certain conditions stated in [Ravenna \(2007\)](#) a finite VAR representation for the vector x_t exists and takes the form

$$x_t = \Phi_1 x_{t-1} + \dots + \Phi_p x_{t-p} + \bar{B}e_t. \quad (\text{A.4})$$

As long as the solution of the DSGE model is stationary, we can always find the $\text{MA}(\infty)$ representation of the vector x_t . Under the assumptions in [Ravenna \(2007\)](#), we can write it as,

$$x_t = \sum_{j=0}^{\infty} F_j \bar{B}e_{t-j}, \quad (\text{A.5})$$

with $F_0 = I$ and $F_j = \bar{A}C^{j-1}D\bar{B}^{-1}$. Using this representation, the cumulative response of the variable in position k of vector x_t , h periods after a shock in position i of vector e_t , is given by

$$\text{CIRF}_{k,i}^M(h) \equiv [F(h) \bar{B}]_{ki}, \quad (\text{A.6})$$

where $F(h) \equiv \sum_{j=0}^h F_j$, and the notation X_{ij} indicates the element in the i th row, j th column of matrix X . Thus, the conditional ERPT after a shock i , for variable k , h periods ahead is given by

$$\text{CERPT}_{k,i}^M(h) \equiv \frac{\text{CIRF}_{k,i}^M(h)}{\text{CIRF}_{\pi_t^S,i}^M(h)},$$

i.e. the ratio of the cumulative response of variable k to the cumulative response of the nominal depreciation (π_t^S), after shock i .

At the same time, if the model (A.1)-(A.2) is the true data generating process, someone using the approach in the VAR-based literature will first estimate a reduced form VAR given by

$$x_t = \Theta_1 x_{t-1} + \dots + \Theta_p x_{t-p} + u_t. \quad (\text{A.7})$$

Clearly, if a finite VAR representation of the DSGE model exists and the lag-length is chosen properly, we have $\Theta_j = \Phi_j$ and $\Omega \equiv E(u_t u_t') = \bar{B} \bar{B}'$. The MA(∞) representation of this reduced-form is

$$x_t = \sum_{j=0}^{\infty} F_j u_{t-j}, \quad (\text{A.8})$$

The Cholesky decomposition of Ω is a matrix P satisfying $\Omega = PP'$. The cumulative IRF of variable k after a shock corresponding to the nominal depreciation equation is given by

$$CIRF_{k,\pi^S}^V(h) \equiv [F(h)P]_{k\pi^S}, \quad (\text{A.9})$$

and the ERPT for variable k , h periods ahead, is computed as,

$$ERPT_k^V(h) \equiv \frac{CIRF_{k,\pi^S}^V(h)}{CIRF_{\pi^S,\pi^S}^V(h)},$$

i.e. the ratio of the cumulative response of variable k to the cumulative response of the nominal depreciation after a shock in the equation of the nominal depreciation.

To study the relationship between $ERPT_k^V(h)$ and $CERPT_{k,i}^M(h)$, assume that the nominal depreciation (π_t^S) is ordered first in the vector x_t .³⁸ Then, we can write the conditional ERPT as

$$CERPT_{k,i}^M(h) = \frac{[F(h)\bar{B}]_{ki}}{[F(h)\bar{B}]_{1i}} = \frac{F(h)_{k1}\bar{B}_{1i} + \dots + F(h)_{km}\bar{B}_{mi}}{F(h)_{11}\bar{B}_{1i} + \dots + F(h)_{1m}\bar{B}_{mi}} = \frac{\sum_{j=1}^m F(h)_{kj}\bar{B}_{ji}}{\sum_{j=1}^m F(h)_{1j}\bar{B}_{ji}}.$$

By the same token, the ERPT from the VAR is

$$ERPT_k^V(h) = \frac{[F(h)P]_{k1}}{[F(h)P]_{11}} = \frac{F(h)_{k1}P_{11} + \dots + F(h)_{km}P_{m1}}{F(h)_{11}P_{11} + \dots + F(h)_{1m}P_{m1}} = \frac{\sum_{j=1}^m F(h)_{kj}P_{j1}}{\sum_{j=1}^m F(h)_{1j}P_{j1}}.$$

In addition, by the properties of the Cholesky decomposition, we have

$$P_{11} = (\Omega_{11})^{1/2}, \quad P_{j1} = \Omega_{j1}(\Omega_{11})^{1/2} \text{ for } j = 2, \dots, m.$$

Thus, the ERPT from the VAR can be written as

$$ERPT_k^V(h) = \frac{F(h)_{k1}\Omega_{11} + \dots + F(h)_{km}\Omega_{m1}}{F(h)_{11}\Omega_{11} + \dots + F(h)_{1m}\Omega_{m1}} = \frac{\sum_{j=1}^m F(h)_{kj}\Omega_{j1}}{\sum_{j=1}^m F(h)_{1j}\Omega_{j1}}.$$

Moreover, as $\Omega = \bar{B}\bar{B}'$, we have

$$\Omega_{ji} = \sum_{s=1}^m \bar{B}_{js}\bar{B}_{is}$$

³⁸This assumption can be relaxed as long as the variables before π_t^S in the VAR vector of variables are strictly exogenous (for instance, in a small and open economy model, foreign variables might appear first in the VAR).

Thus,

$$\begin{aligned}
ERPT^V(h) &= \frac{\sum_{j=1}^m F(h)_{kj} (\sum_{s=1}^m \bar{B}_{js} \bar{B}_{1s})}{\sum_{j=1}^m F(h)_{1j} (\sum_{s=1}^m \bar{B}_{js} \bar{B}_{1s})} = \frac{\sum_{s=1}^m \left(\sum_{j=1}^m F(h)_{kj} \bar{B}_{js} \right) \bar{B}_{1s}}{\sum_{s=1}^m \left(\sum_{j=1}^m F(h)_{1j} \bar{B}_{js} \right) \bar{B}_{1s}} \\
&= \frac{\sum_{s=1}^m CIRF_{k,s}^M(h) \bar{B}_{1s}}{\sum_{s=1}^m CIRF_{1,s}^M(h) \bar{B}_{1s}} = \frac{\sum_{s=1}^m CERPT_{k,s}^M(h) CIRF_{1,s}^M(h) \bar{B}_{1s}}{\sum_{s=1}^m CIRF_{1,s}^M(h) \bar{B}_{1s}} \\
&= \sum_{s=1}^m CERPT_{k,s}^M(h) \omega_s(h),
\end{aligned}$$

where $\omega_s(h) \equiv \frac{CIRF_{1,s}^M(h) \bar{B}_{1s}}{\sum_{s=1}^m CIRF_{1,s}^M(h) \bar{B}_{1s}}$.

To grasp some intuition on the weight $\omega_s(h)$, notice that at $h = 0$,

$$\omega_s(h) \equiv \frac{(\bar{B}_{1s})^2}{\sum_{s=1}^m (\bar{B}_{1s})^2},$$

i.e. the fraction of the one-step-ahead forecast-error-variance of the nominal exchange rate that is due to the shock s . In other words, the weight of the conditional ERPT given shock s depends on how much of the fluctuations in the nominal exchange rate is explained by this shock. For $h \geq 1$, the forecast-error variance is adjusted by the ratio of the response of the NER at period h relative to that at $h = 0$.

A.2 ERPT from the Population VAR

From the linearized solution of the DSGE model (5), provided stationarity, the variance-covariance matrix $\Sigma_0 \equiv E(y_t y_t')$ satisfies,

$$\Sigma_0 = F \Sigma_0 F' + Q Q', \quad (\text{A.10})$$

which can be easily computed.³⁹ In addition, the matrix containing the auto-covariance of order p is $\Sigma_p \equiv E(y_t y_{t-p}') = F^p \Sigma_0$ for $p > 0$. Finally, we are interested in subset x_t of n variables from y_t , that will be included in the VAR model, defined as $x_t \equiv S y_t$ for an appropriate choice of S . In that case, we have

$$E(x_t x_{t-p}') = S E(y_t y_{t-p}') S' = S \Sigma_p S'. \quad (\text{A.11})$$

for $p \geq 0$.

The structural VAR(p) model for the vector x_t in (2)-(3) can be written in more compact form, defining the vector $X_t = [x_t' \ x_{t-1}' \ \dots \ x_{t-p+1}']'$, in two alternative ways. Either,

$$x_t = \Phi X_{t-1} + P w_t, \quad (\text{A.12})$$

where $\Phi = [\Phi_1 \ \dots \ \Phi_p]$ or,

$$X_t = \tilde{\Phi} X_{t-1} + U_t, \quad (\text{A.13})$$

where,

$$\tilde{\Phi} = \begin{bmatrix} \Phi & \\ I_{n(p-1)} & 0_{n(p-1) \times n} \end{bmatrix}, \quad U_t = \tilde{P} w_t, \quad \tilde{P} = \begin{bmatrix} P \\ 0_{n(p-1) \times n} \end{bmatrix}.$$

Using (A.13) the IRF of the variable in position j of vector x_t to the shock associated with the variable in position i of the same vector, h periods after the shock, is given by the $\{j, i\}$ element of the matrix $\tilde{\Phi}^h \tilde{P}$. The cumulative IRF is the element $\{j, i\}$ of matrix $\sum_{s=0}^h \tilde{\Phi}^s \tilde{P}$.

An econometrician would proceed by choosing a lag order p in the VAR and estimate (A.12) by OLS. If she had an infinite sample available, she could estimate (A.12) using the population OLS; i.e. choosing $\hat{\Phi}$ to minimize,

$$E \left[(x_t - \hat{\Phi} X_{t-1})' (x_t - \hat{\Phi} X_{t-1}) \right].$$

This is equivalent to $\hat{\Phi}$ satisfying the first order condition,

$$E \left[(x_t - \hat{\Phi} X_{t-1}) X_{t-1}' \right] = 0,$$

which can be solved to obtain,

$$\hat{\Phi} = E(x_t X_{t-1}') [E(X_{t-1} X_{t-1}')]^{-1}. \quad (\text{A.14})$$

Similarly,

$$\begin{aligned} \hat{\Omega} &= E(u_t u_t') = E \left[(x_t - \hat{\Phi} X_{t-1}) (x_t - \hat{\Phi} X_{t-1})' \right] \\ &= E(x_t x_t') + \hat{\Phi} E(X_{t-1} X_{t-1}') \hat{\Phi}' - E(x_t X_{t-1}') \hat{\Phi}' - \hat{\Phi} E(X_{t-1} x_t') \\ &= E(x_t x_t') + E(x_t X_{t-1}') [E(X_{t-1} X_{t-1}')]^{-1} E(X_{t-1} x_t') - \\ &\quad E(x_t X_{t-1}') [E(X_{t-1} X_{t-1}')]^{-1} E(X_{t-1} x_t') - E(x_t X_{t-1}') [E(X_{t-1} X_{t-1}')]^{-1} E(X_{t-1} x_t') \\ &= E(x_t x_t') - E(x_t X_{t-1}') [E(X_{t-1} X_{t-1}')]^{-1} E(X_{t-1} x_t') = E(x_t x_t') - \hat{\Phi} E(X_{t-1} x_t') \end{aligned} \quad (\text{A.15})$$

³⁹For instance, $\text{vec}(\Sigma_0) = (I - F \otimes F)^{-1} \text{vec}(Q Q')$.

In most applied cases, with finite samples, econometricians estimate the parameters of the VAR and use asymptotic theory to derive probability limits and limiting distributions to perform inference,⁴⁰ such as hypothesis testing or computing confidence bands. The case we want to analyze here is different, as we assume the DSGE model is the true data generating process, and we wish to compute the model that an econometrician would estimate with an infinite or population sample. This is equivalent to compute $\hat{\Phi}$ and $\hat{\Omega}$ in (A.14)-(A.15) using the population moments from the DSGE.

Given $x_t = Sy_t$, and recalling the definition of X_t , we have,

$$\begin{aligned}
E(x_t x_t') &= S \Sigma_0 S', \\
E(x_t X_{t-1}') &= [E(x_t x_{t-1}') \ E(x_t x_{t-2}') \ \dots \ E(x_t x_{t-p}')] = [S \Sigma_1 S' \ S \Sigma_2 S' \ \dots \ S \Sigma_p S'] \\
E(X_{t-1} X_{t-1}') &= \begin{bmatrix} E(x_{t-1} x_{t-1}') & E(x_{t-1} x_{t-2}') & \dots & E(x_{t-1} x_{t-p}') \\ E(x_{t-2} x_{t-1}') & E(x_{t-2} x_{t-2}') & \dots & E(x_{t-2} x_{t-p}') \\ \vdots & \vdots & \ddots & \vdots \\ E(x_{t-p} x_{t-1}') & E(x_{t-p} x_{t-2}') & \dots & E(x_{t-p} x_{t-p}') \end{bmatrix} = \\
&= \begin{bmatrix} E(x_t x_t') & E(x_t x_{t-1}') & \dots & E(x_t x_{t-p+1}') \\ E(x_{t-1} x_t') & E(x_{t-1} x_{t-1}') & \dots & E(x_{t-1} x_{t-p+2}') \\ \vdots & \vdots & \ddots & \vdots \\ E(x_{t-p+1} x_t') & E(x_{t-p+2} x_t') & \dots & E(x_{t-p+1} x_{t-p+1}') \end{bmatrix} = \begin{bmatrix} S \Sigma_0 S' & S \Sigma_1 S' & \dots & S \Sigma_{p-1} S' \\ S \Sigma_1' S' & S \Sigma_0 S' & \dots & S \Sigma_{p-2} S' \\ \vdots & \vdots & \ddots & \vdots \\ S \Sigma_{p-1}' S' & S \Sigma_{p-2}' S' & \dots & S \Sigma_0 S' \end{bmatrix}
\end{aligned}$$

which are all the elements required to compute $\hat{\Phi}$ and $\hat{\Omega}$.

A final comment relating the usual practice in the VAR literature. In most papers the vector x_t contains foreign variables. If the assumption of a small and open economy is used, it is generally assumed that the matrices Φ_j for $j = 1, \dots, p$ are block lower triangular: i.e. lags of domestic variables cannot affect foreign variables. In practice, this second constraint is implemented by estimating the matrices Φ_j by FGLS or FIML, applying the required restrictions. Here, however, if the DSGE model assumes that foreign variables cannot be affected by domestic variables, the auto-covariance matrices Σ_j will have zeros in the appropriate places, so that $\hat{\Phi}$ will display the same zero constraints that an econometrician would impose.

⁴⁰For instance, (A.14) and (A.15) are the probability limits of the OLS estimators for Φ and Ω , by virtue of both the Law of Large Numbers and the Continuous Mapping Theorem.

B Simple DSGE Model Appendix

B.1 Optimality Conditions

B.1.1 Household

From the decision of final consumption, labor and bonds, and defining as λ_t the multiplier of the budget constraint, we have the first order conditions:

$$\begin{aligned} C_t^{-\sigma} - P_t \lambda_t &= 0 \\ -\xi(h_t)^\varphi + W_t \lambda_t &= 0 \\ -\lambda_t + \beta E_t \lambda_{t+1} R_t &= 0 \\ -\lambda_t S_t + \beta E_t \lambda_{t+1} S_{t+1} R_t^* &= 0 \end{aligned}$$

In addition, the optimality conditions for the decision between tradable and non-tradable consumption are:

$$\begin{aligned} C_t^N &= \gamma \left(\frac{P_t^N}{P_t} \right)^{-\varrho} C_t \\ C_t^T &= (1 - \gamma) \left(\frac{P_t^T}{P_t} \right)^{-\varrho} C_t \end{aligned}$$

B.1.2 Firms in N Sector

The aggregation creates a Δ variable in this case:

$$\begin{aligned} h_t &= \int_0^1 h_t(i) di = \Delta_t^{Nh} (Y_t^N)^{\frac{1}{1-\alpha}} \\ \Delta_t^{Nh} &= \int_0^1 \left(\frac{P_t(i)^N}{P_t^N} \right)^{-\frac{\epsilon}{1-\alpha}} di \end{aligned}$$

The problem solved by firms when choosing prices can be written as:

$$\begin{aligned} \max L = E_t \sum_{\tau=0}^{\infty} (\beta \theta)^\tau \Lambda_{t,t+\tau} &\left[\frac{P_t^N(i)^{1-\epsilon}}{(P_{t+\tau}^N)^{-\epsilon}} Y_{t+\tau}^N \left[\prod_{s=1}^{\tau} (\pi_{t+s-1})^\zeta \bar{\pi}^{1-\zeta} \right]^{1-\epsilon} - \right. \\ &\left. P_t^N(i)^{\frac{-\epsilon}{1-\alpha}} W_{t+\tau} \left[\frac{Y_{t+\tau}^N}{(P_{t+\tau}^N)^{-\epsilon}} \right]^{\frac{1}{1-\alpha}} \left[\prod_{s=1}^{\tau} (\pi_{t+s-1})^\zeta \bar{\pi}^{1-\zeta} \right]^{\frac{-\epsilon}{1-\alpha}} \right] \end{aligned}$$

with $\Lambda_{t,t+\tau}$ the stochastic discount factor. Defining $P_t^{N,*}$ as the optimal price chosen by the firms, the FOC can be simplified and written recursively as the following system of two equations:

$$\begin{aligned}
f_t^N &= \frac{\epsilon - 1}{\epsilon} (P_t^{N,*})^{1-\epsilon} \frac{Y_t^N}{(P_t^N)^{-\epsilon}} + \beta \theta E_t \left(\frac{P_t^{N,*}}{P_{t+1}^{N,*}} \right)^{1-\epsilon} \Lambda_{t,t+1} \left[(\pi_t)^\zeta \bar{\pi}^{1-\zeta} \right]^{1-\epsilon} f_{t+1}^N \\
f_t^N &= \frac{1}{1-\alpha} (P_t^{N,*})^{-\frac{\epsilon}{1-\alpha}} W_t \left[\frac{Y_t^N}{(P_t^N)^{-\epsilon}} \right]^{\frac{1}{1-\alpha}} + \beta \theta E_t \Lambda_{t,t+1} \left(\frac{P_t^{N,*}}{P_{t+1}^{N,*}} \right)^{-\frac{\epsilon}{1-\alpha}} \\
&\quad \left[\left(\pi_t^{1-\varrho} \right)^\zeta \bar{\pi}^{1-\zeta} \right]^{-\frac{\epsilon}{1-\alpha}} f_{t+1}^N
\end{aligned}$$

B.1.3 Market Clearing

All markets clear:

$$\begin{aligned}
B_t &= 0 \\
Y_t^N &= \Delta_t^N C_t^N
\end{aligned}$$

Which correspond to the local bonds market and goods market. The Δ_t^N variable is a measure of price dispersion in N , defined as:

$$\Delta_t^N = \int_0^1 \left(\frac{P_t^N(i)}{P_t^N} \right)^{-\epsilon} di$$

The rest of the equations correspond to policy and foreign equations described in the text and to equations concerning the evolution of price indexes. In addition, we have the resource constraint:

$$S_t B_t^* = S_t P_t^{T,*} (Y_t^T - C_t^T) + S_t R_{t-1}^* B_{t-1}^*$$

And definitions of trade balance and real and nominal GDP:

$$\begin{aligned}
TB_t &= P_t^T (Y_t^T - C_t^T) \\
GDP_t &= C_t + Y_t^T - C_t^T \\
P_t^Y GDP_t &= P_t C_t + TB_t
\end{aligned}$$

B.2 Equilibrium Conditions

This sections describes the equilibrium conditions after the variables were redefined to make them stationary. The transformations made to the variables were: all lower case prices are the corresponding capital price divided by the CPI Index, with the exception of $p_t^{N,*} = P_t^{N,*}/P_t^N$, all inflation definitions are the corresponding price index divided by the price index in the previous period. And particular definitions are $\tilde{\lambda}_t = \lambda_t P_t$, $b_t^* = B_t^*/P_t^T$, $tb_t = TB_t/P_t$, $\tilde{f}_t^N = f_t^N/P_t^N$.

There are 22 endogenous variables,

$$\{C_t, \tilde{\lambda}_t, h_t, w_t, R_t^*, \pi_t^S, \pi_t, R_t, C_t^N, C_t^T, p_t^N, p_t^T, \Delta_t^{Nh}, Y_t^N, p_t^{N,*}, \pi_t^N, \tilde{f}_t^N, GDP_t, b_t^*, \Delta_t^N, p_t^Y, tb_t\}$$

and 3 shocks $\{\epsilon_t^m, \pi_t^*, R_t^W\}$.

$$C_t^{-\sigma} = \tilde{\lambda}_t \quad (\text{B.1})$$

$$\chi(h_t)^\varphi = \tilde{\lambda}_t w_t \quad (\text{B.2})$$

$$\tilde{\lambda}_t = \beta E_t \frac{\tilde{\lambda}_{t+1} R_t^* \pi_{t+1}^S}{\pi_{t+1}} \quad (\text{B.3})$$

$$\tilde{\lambda}_t = \beta E_t \frac{\tilde{\lambda}_{t+1} R_t}{\pi_{t+1}} \quad (\text{B.4})$$

$$C_t^N = \gamma (p_t^N)^{-\varrho} C_t \quad (\text{B.5})$$

$$C_t^T = (1 - \gamma) (p_t^T)^{-\varrho} C_t \quad (\text{B.6})$$

$$1 = (1 - \gamma) (p_t^T)^{1-\varrho} + \gamma (p_t^N)^{1-\varrho} \quad (\text{B.7})$$

$$h_t = \Delta_t^{Nh} (Y_t^N)^{\frac{1}{1-\alpha}} \quad (\text{B.8})$$

$$\Delta_t^{Nh} = (1 - \theta) (p_t^{*,N})^{-\frac{\epsilon}{1-\alpha}} + \theta \left(\frac{(\pi_{t-1})^\zeta \bar{\pi}^{1-\zeta}}{\pi_t^N} \right)^{-\frac{\epsilon}{1-\alpha}} \Delta_{t-1}^{Nh} \quad (\text{B.9})$$

$$\begin{aligned} \tilde{f}_t^N = & \frac{\epsilon - 1}{\epsilon} (p_t^{N,*})^{1-\epsilon} Y_t^N + \\ & \beta \theta E_t \left(\frac{p_t^{N,*}}{p_{t+1}^{N,*}} \frac{1}{\pi_{t+1}^N} \right)^{1-\epsilon} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \left[(\pi_t)^\zeta \bar{\pi}^{1-\zeta} \right]^{1-\epsilon} \frac{\pi_{t+1}^N}{\pi_{t+1}} \tilde{f}_{t+1}^N \end{aligned} \quad (\text{B.10})$$

$$\begin{aligned} \tilde{f}_t^N = & \frac{1}{1-\alpha} (p_t^{N,*})^{-\frac{\epsilon}{1-\alpha}} \frac{w_t}{p_t^N} (Y_t^N)^{\frac{1}{1-\alpha}} + \\ & \beta \theta E_t \left(\frac{p_t^{N,*}}{p_{t+1}^{N,*}} \frac{1}{\pi_{t+1}^N} \right)^{-\frac{\epsilon}{1-\alpha}} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \left[(\pi_t)^\zeta \bar{\pi}^{1-\zeta} \right]^{-\frac{\epsilon}{1-\alpha}} \frac{\pi_{t+1}^N}{\pi_{t+1}} \tilde{f}_{t+1}^N \end{aligned} \quad (\text{B.11})$$

$$\pi_t^N = \frac{p_t^N}{p_{t-1}^N} \pi_t \quad (\text{B.12})$$

$$1 = (1 - \theta) (p_t^{*,N})^{1-\epsilon} + \theta \left[(\pi_{t-1})^\zeta \bar{\pi}^{1-\zeta} \right]^{1-\epsilon} \left(\frac{1}{\pi_t^N} \right)^{1-\epsilon} \quad (\text{B.13})$$

$$\left(\frac{R_t}{R} \right) = \left(\frac{\pi_t}{\bar{\pi}} \right)^{\alpha_\pi} \left(\frac{GDP_t}{GDP} \right)^{\alpha_{gdp}} e^{\epsilon_t^m} \quad (\text{B.14})$$

$$\frac{p_t^T}{p_{t-1}^T} = \frac{\pi_t^S \pi_t^*}{\pi_t} \quad (\text{B.15})$$

$$R_t^* = R_t^W + \phi_B \left(e^{\bar{b} - b_t^*} - 1 \right) \quad (\text{B.16})$$

$$Y_t^N = \Delta_t^N C_t^N \quad (\text{B.17})$$

$$\Delta_t^N = (1 - \theta) \left(p_t^{*,N} \right)^{-\epsilon} + \theta \left(\frac{(\pi_{t-1})^\zeta \bar{\pi}^{1-\zeta}}{\pi_t^N} \right)^{-\epsilon} \Delta_{t-1}^N \quad (\text{B.18})$$

$$tb_t = p_t^T (Y^T - C_t^T) \quad (\text{B.19})$$

$$p_t^T b_t^* = tb_t + \frac{p_t^T}{\pi_t^*} R_{t-1}^* b_{t-1}^* \quad (\text{B.20})$$

$$GDP_t = C_t + Y^T - C_t^T \quad (\text{B.21})$$

$$p_t^Y GDP_t = C_t + tb_t \quad (\text{B.22})$$

And the equations for the exogenous processes that are described in the text.

B.3 Steady state

The given endogenous are $\{h, p^T, s^{tb}\}$ and the exogenous variables or parameters calculated are $\{\pi^*, \xi, y^T\}$.

From (B.16)

$$R^* = R^W$$

from (B.14)

$$\pi = \bar{\pi}$$

from (B.4)

$$R = \pi / \beta$$

from (B.3)

$$\pi^s = \pi / (\beta R^*)$$

from (B.12)

$$\pi^N = \pi$$

from (B.13)

$$p^{*,N} = 1$$

from (B.9), (B.18)

$$\Delta^{Nh} = \Delta^N = 1$$

from (B.15)

$$\pi^* = \pi / \pi^S$$

from (B.7)

$$p^N = \left(\frac{1 - (1 - \gamma)(p^T)^{1-\varrho}}{\gamma} \right)^{\frac{1}{1-\varrho}}$$

from (B.8)

$$Y^N = h^{1-\alpha}$$

from (B.10)

$$\tilde{f}^N = \frac{\epsilon - 1}{\epsilon} (p^{N,*})^{1-\epsilon} Y^N \frac{1}{1 - \beta\theta}$$

from (B.11)

$$w = \tilde{f}^N (1 - \beta\theta) (1 - \alpha) p^N \frac{1}{(p^{N,*})^{-\frac{\epsilon}{1-\alpha}} (Y^N)^{\frac{1}{1-\alpha}}}$$

from (B.17)

$$C^N = Y^N$$

from (B.5)

$$C = C^N (p^N)^\varrho / \gamma$$

from (B.6)

$$C^T = (1 - \gamma) (p^T)^{-\varrho} C$$

from (B.1)

$$\tilde{\lambda} = C^{-\sigma}$$

from (B.2)

$$\chi = \tilde{\lambda} w / h^\varphi$$

from (B.22)

$$p^Y GDP = C / (1 - s^{tb})$$

$$tb = s^{tb} p^Y GDP$$

from (B.19)

$$y^T = \frac{tb}{p_t^T} + C^T$$

from (B.20)

$$b^* = \frac{tb}{p_t^T (1 - R^* / \pi^*)}$$

from (B.21)

$$GDP = C + Y^T - C^T$$

Finally

$$p^Y = \frac{p^Y GDP}{GDP}$$

$$\bar{b} = b^*$$

C Additional IRFs Baseline Model

Figure 16: IRFs to Monetary Shock for Alternative ρ_{ϵ^m}

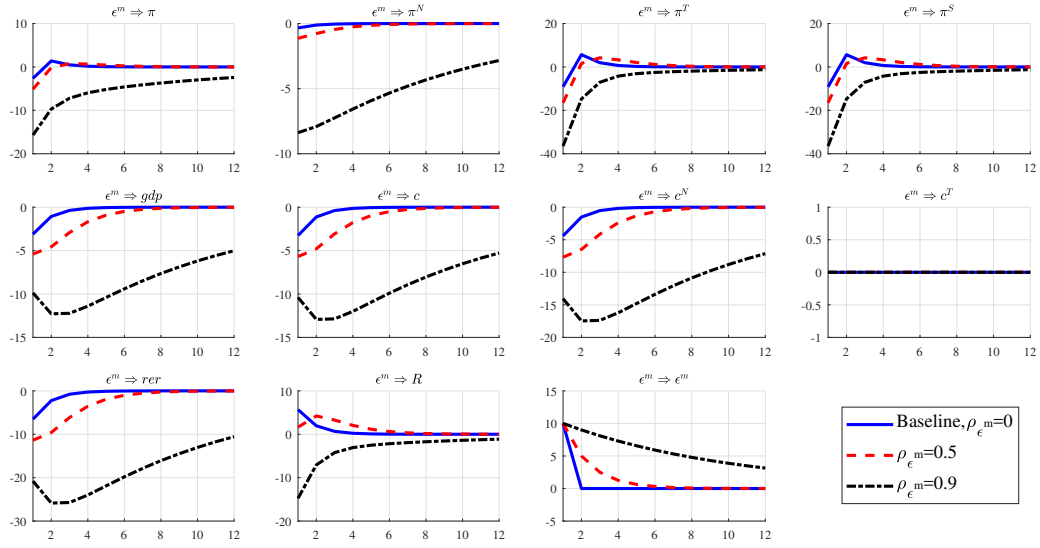


Figure 17: IRFs to External Inflation for Alternative periods with Fixed Interest Rate

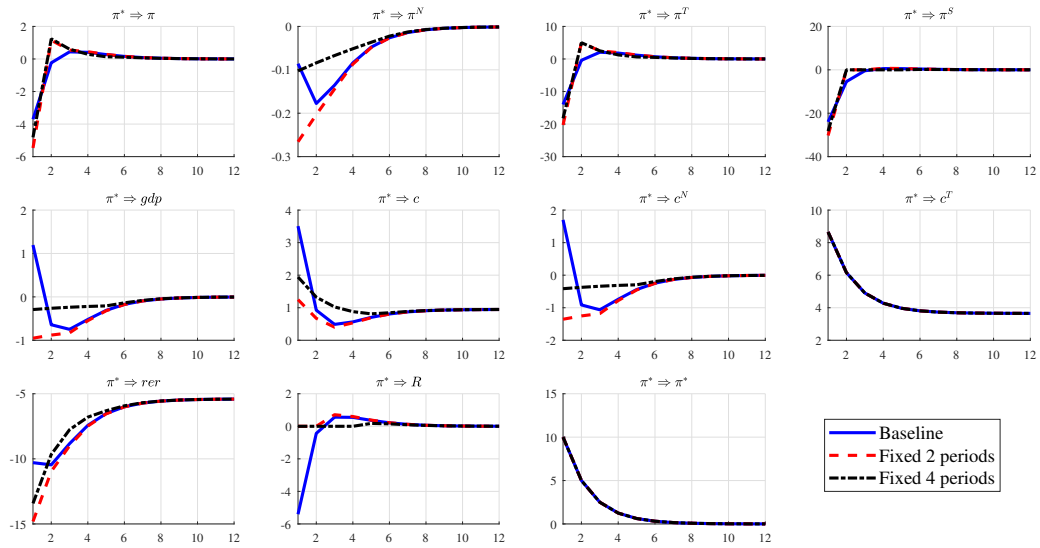


Figure 18: IRFs to External Interest Rate for Alternative periods with Fixed Interest Rate

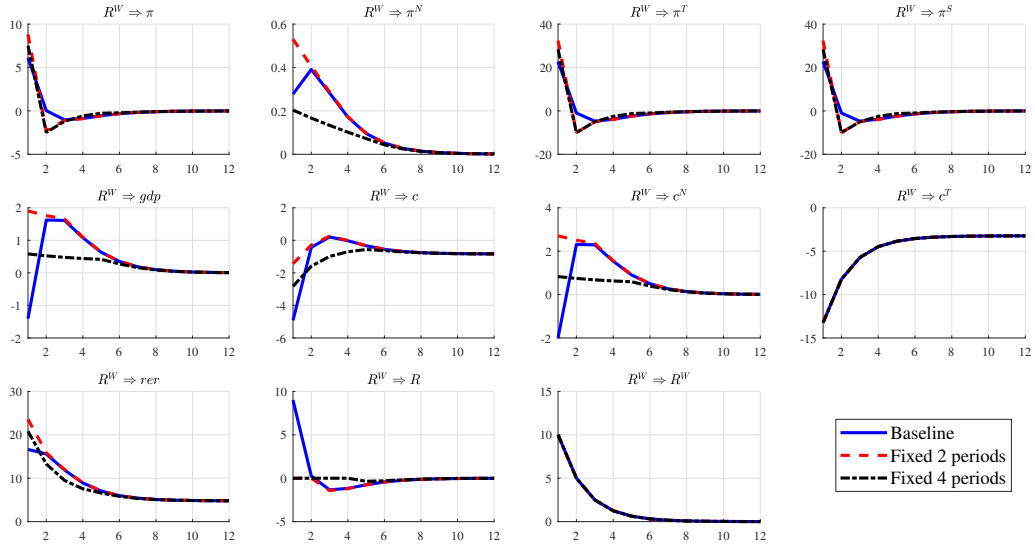


Figure 19: IRFs to Monetary Shock for Alternative Indexation

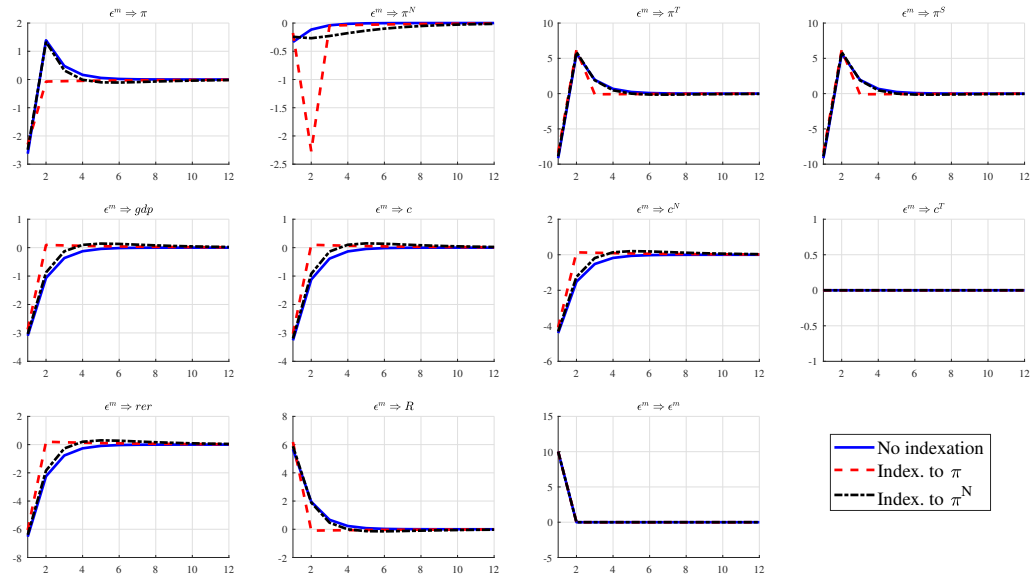


Figure 20: IRFs to External Inflation for Alternative Indexation

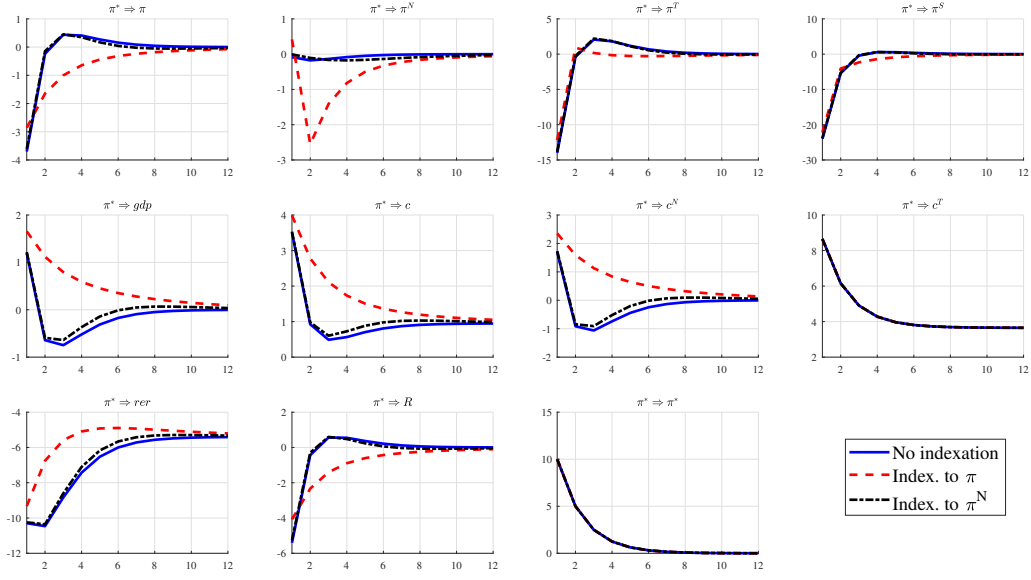
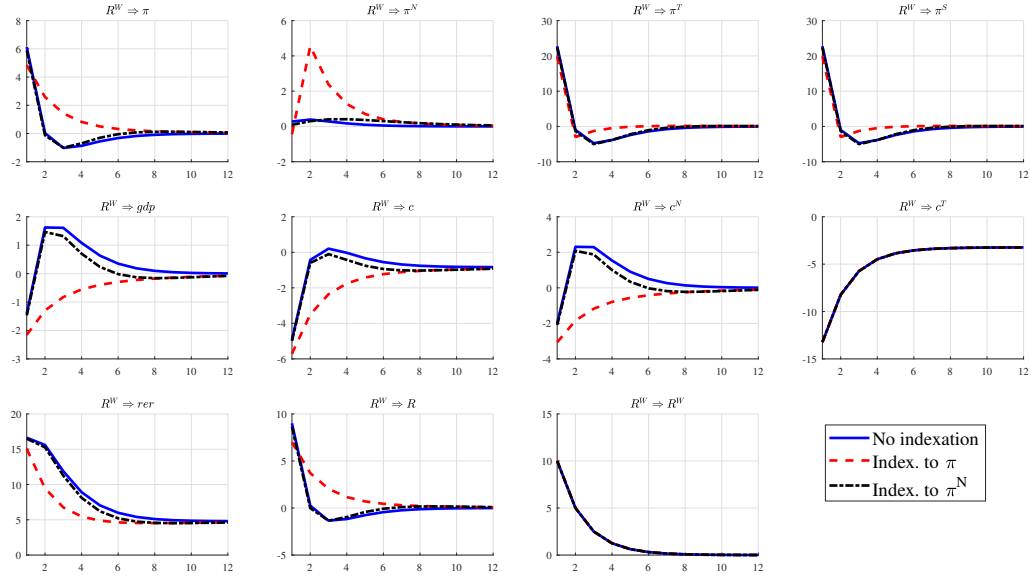


Figure 21: IRFs to External Interest Rate for Alternative Indexation



D Quantitative DSGE Model Appendix

This appendix has four sections. The first presents all agents in the model, their optimization problems and constraints, as well as the driving forces. The second describes the parametrization strategy and studies the goodness of fit of the model. The third derives the optimality conditions for the different agents. The final section presents the equilibrium conditions and the computation of the steady state.

D.1 Model description

D.1.1 Households

There is a representative household that consumes, works, saves, invests and rents capital to the producing sectors. Her goal is to maximize,

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t^\beta \left\{ \frac{(C_t - \phi_c \tilde{C}_{t-1})^{1-\sigma}}{1-\sigma} - \kappa_t \left(\xi_t^{h,X} \frac{h_t^{X^{1+\varphi}}}{1+\varphi} + \xi_t^{h,N} \frac{h_t^{N^{1+\varphi}}}{1+\varphi} \right) \right\}$$

where C_t is consumption and h_t^J for $J = \{X, N\}$ are hours worked in sector J . \tilde{C}_t denotes aggregate consumption (i.e. the utility exhibits external habits),⁴¹ and $\kappa_t \equiv (\tilde{C}_t - \phi_c \tilde{C}_{t-1})^{-\sigma}$.⁴² There are three preference shocks, ξ_t^β and $\xi_t^{h,J}$ for $J = \{X, N\}$: the former affects inter-temporal decisions, while the latter is a labor supply shifter in sector $J = \{X, N\}$. The parameters are given by β (the discount factor), ϕ_c (external habits), σ (risk aversion) and φ (the inverse of the Frisch elasticity of labor supply).

The budget constraint is

$$P_t C_t + S_t B_t^* + B_t + P_t^I I_t^N + P_t^I I_t^X = S_t R_{t-1}^* B_{t-1}^* + R_{t-1} B_{t-1} + h_t^{X,d} \int_0^1 W_t^X(j) \left(\frac{W_t^X(j)}{W_t^X} \right)^{-\epsilon_W} dj \\ + h_t^{N,d} \int_0^1 W_t^N(j) \left(\frac{W_t^N(j)}{W_t^N} \right)^{-\epsilon_W} dj + P_t^N R_t^N K_{t-1}^N + P_t^X R_t^X K_{t-1}^X + T_t + \Pi_t.$$

Here P_t the price of the consumption good, S_t the exchange rate, B_t^* the amount of external bonds bought by the household in period t , B_t amount of local bonds bought by the household in t , P_t^I is the price of the investment good, I_t^J is investment in capital of the sector J , $h_t^{J,d}$ is labor demand in sector J , W_t^J is the wage index in sector J , $W_t(j)$ is the wage of variety i in sector J , R_t^* is the external interest rate, R_t is the internal interest rate, R_t^J is the real rate from renting capital to firms in sector J , K_t^J is capital specific for sector J , P_t^J is the price of goods J , T_t are transfers made by the government and finally Π_t has all the profits of the firms in all sectors. The parameter ϵ_W is the elasticity of substitution among varieties of labor.

The formulation of the wage-setting problem follows [Schmitt-Grohé and Uribe \(2006\)](#). In this setup, households supply a homogeneous labor input that is transformed by monopolistically competitive labor unions into a differentiated labor input. The union takes aggregate variables as given and decides the nominal wage, while supplying enough labor to meet the demand in each market. The wage of each differentiated labor input is chosen optimally each period with a constant probability $1 - \theta_{WJ}$ for $J = \{X, N\}$. When wages cannot be freely chosen they are updated by $(\pi_{t-1})^{\zeta_{WJ}} \bar{\pi}^{1-\zeta_{WJ}}$, with $\zeta_{WJ} \in [0, 1]$, π_{t-1} denoting previous-period CPI inflation and $\bar{\pi}$ the inflation target set by the

⁴¹In equilibrium $\tilde{C}_t = C_t$.

⁴²This utility specification follows [Galí et al. \(2012\)](#), and it is designed to eliminate the wealth effect on the supply of labor while keeping separability between consumption and labor.

Central Bank.

D.1.2 Consumption Goods

Consumption C_t is composed by three elements: core consumption (C_t^{NFE}), food (C_t^F) and energy (C_t^E). For simplicity, food and energy consumption are assumed exogenous and normalized to one (so total and core consumption are equal). In contrast the price of the consumption good will be a composite of the price of the core good, energy and food the following way:

$$P_t = (P_t^{NFE})^{1-\gamma_{FC}-\gamma_{EC}} (P_t^F)^{\gamma_{FC}} (P_t^E)^{\gamma_{EC}}$$

Where P_t^{NFE} is the price of core consumption, P_t^F is the price of food and P_t^E is the price of energy.⁴³ We further assume that the prices of both F and E relative to that of the tradable composite (T , defined below) follow exogenous processes (p_t^F and p_t^E respectively).⁴⁴

Core consumption is a composite of non-tradable consumption C_t^N and tradable consumption C_t^T , while the latter is composed by exportable C_t^X and importable C_t^M goods,

$$\begin{aligned} C_t^{NFE} &= \left[\gamma^{1/\varrho} (C_t^N)^{\frac{\varrho-1}{\varrho}} + (1-\gamma)^{1/\varrho} (C_t^T)^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}} \\ C_t^T &= \frac{(C_t^X)^{\gamma_T} (C_t^M)^{(1-\gamma_T)}}{(1-\gamma_T)^{(1-\gamma_T)} \gamma_T^{\gamma_T}} \\ C_t^J &= \int_0^1 G(C_t^J(i), \xi_t^J) di, \end{aligned}$$

where ϱ is the elasticity of substitution between non-tradables and tradables. The last equation specifies that exportable, importable and non-tradable consumption are made of a continuum of differentiated goods in each sector, combined by an aggregator G , which we assume features a constant elasticity of substitution $\epsilon_J > 1$ for $J = \{X, M, N\}$. Moreover, it is assumed that the aggregator is subject to exogenous disturbances (ξ_t^J), generating markup-style shocks in the pricing decisions by firms as in [Smets and Wouters \(2007\)](#).

D.1.3 Capital and Investment Goods

The evolution of the capital stock in sector J is given by

$$K_t^J = \left[1 - \Gamma \left(\frac{I_t^J}{I_{t-1}^J} \right) \right] u_t I_t^J + (1 - \delta) K_{t-1}^J,$$

for $J = \{X, N\}$. It is assumed that installed capital is sector-specific, there are adjustment costs to capital accumulation with $\Gamma'(\cdot) > 0$ and $\Gamma''(\cdot) > 0$ and there is a shock u_t to the marginal efficiency of investment.⁴⁵ The parameter $\delta \in (0, 1)$ is the depreciation rate.

Households choose how much to invest in each type of capital, which constitutes the demand for investment. The supply of investment is assumed to be provided by competitive firms that have a technology similar to the consumption preferences of households, but with different weights, γ_I and

⁴³The goal of this simplified specification is to be able to separate the dynamics of core an total inflation, without complicating significantly the supply side of the model.

⁴⁴The implicit assumption is that food and energy are made of tradable goods, although not all of them are strictly imported. This assumption is reasonable given the Chilean production structure of these goods.

⁴⁵We assume that u_t is the same for both sectors, as we do not have data on sectoral investment at a quarterly frequency.

γ_{TI} , and elasticity of substitution ϱ_I :

$$I_t = \left[\gamma_I^{1/\varrho_I} (\tilde{I}_t^N)^{\frac{\varrho_I-1}{\varrho_I}} + (1 - \gamma_I)^{1/\varrho_I} (\tilde{I}_t^T)^{\frac{\varrho_I-1}{\varrho_I}} \right]^{\frac{\varrho_I}{\varrho_I-1}}$$

$$\tilde{I}_t^T = \frac{(\tilde{I}_t^X)^{\gamma_{TI}} (\tilde{I}_t^M)^{1-\gamma_{TI}}}{(1 - \gamma_{TI})^{(1-\gamma_{TI})} \gamma_{TI}^{\gamma_{TI}}}$$

Similar to consumption, each investment \tilde{I}_t^J for $J = \{X, M, N\}$ is a continuum of the differentiated goods in each sector with the same aggregator G .

D.1.4 Firms

There are three sectors in addition to commodities (assumed to be an endowment); exportable, importable and non-tradable. Firms in the importable sector buy an homogeneous good from foreigners and differentiate it, creating varieties which are demanded by households and firms. Firms in the exportable and non-tradable sector combine a value added created using labor and capital with a composite of the varieties sold by the importable sector to produce their final product.

Each firm in each sector supplies a differentiated product, generating monopolistic power. Given their marginal cost, they maximize prices a la Calvo with probability θ_J for $J = \{X, M, N\}$ of not being able to choose their price optimally each period. When not chosen optimally, the price is updated according to: $[(\pi_{t-1}^J)^{\varrho_J} (\pi_{t-1})^{1-\varrho_J}]^{\zeta_J} \bar{\pi}^{1-\zeta_J}$, with π_{t-1}^J being inflation of sector J in the previous period, and parameters $\{\varrho_J, \zeta_J\} \in [0, 1]$. In this way, the indexation specification is flexible enough to accommodate both dynamic as well as static indexation, with a backward-looking feedback that can be related to either sector specific or aggregate inflation; and we let the data tell the preferred values for ϱ_J and ζ_J in each sector.

1. Sector M:

Each firm i in this sector produces a differentiated product from an homogeneous foreign input with the technology $Y_t^M(i) = M_t(i)$. The price of their input is given by $P_{m,t} = S_t P_t^{M*}$, where $P_{m,t}$ is the price of the good that is imported in local currency and P_t^{M*} is the price in foreign currency and is exogenously given.

2. Sector X and N:

All firms in both sectors have the same format. Each firm i of sector J produces a differentiated product that is a combination of value added $V_t^J(i)$ and an importable input $M_t^J(i)$, which is a combination of a continuum of the goods sold by M sector and energy. They have the technology,

$$Y_t^J(i) = (V_t^J(i))^{\gamma_J} (M_t^J(i))^{1-\gamma_J},$$

with $\gamma_J \in [0, 1]$ and value added is produced by,

$$V_t^J(i) = z_t^J [K_{t-1}^J(i)]^{\alpha_J} [A_t^J h_t^{J,d}(i)]^{1-\alpha_J}.$$

with $\alpha_J \in [0, 1]$, z_t^J is a stationary technology shock and A_t^J is a non-stationary stochastic trend in technology. To maintain a balance-growth path, we assume that both trends co-integrate in the long-run. In particular, we assume that $a_t \equiv A_t^N/A_{t-1}^N$ is an exogenous process and A_t^X evolves according to,

$$A_t^X = (A_{t-1}^X)^{1-\Gamma_X} (A_t^N)^{\Gamma_X}$$

The factor demand for these firms can be solved in two stages:

- (a) Optimal production of $V_t^J(i)$: Firms are price takers, so they choose the optimal combination of capital and labor to minimize their cost,

$$\min_{K_{t-1}^J(i), h_t^J(i)} P_t^J R_t^J K_{t-1}^J(i) + W_t^J h_t^J(i) + \mu \left\{ V_t^J(i) - z_t^J [K_{t-1}^J(i)]^{\alpha_J} [A_t^J h_t^{J,d}(i)]^{1-\alpha_J} \right\}$$

- (b) Optimal production of $Y_t^J(i)$: Firms choose the optimal combination of value added and imported inputs to minimize their cost,

$$\min_{M_t^J(i), V_t^J(i)} MC_t^{VJ} V_t^J(i) + P_t^{ME} M_t^J(i) + \mu \{ Y_t^J(i) - [V_t^J(i)]^{\gamma_J} [M_t^J(i)]^{1-\gamma_J} \}$$

where MC_t^{VJ} is the marginal cost of producing $V_t^J(i)$, which is the same for all firms, and P_t^{ME} is the price of a composite between a continuum of the importable goods sold by the M sector and energy, i.e.

$$P_t^{ME} = (P_t^M)^{1-\gamma_{EM}} (P_t^E)^{\gamma_{EM}}$$

with $\gamma_{EM} \in [0, 1]$. As in the case of the household with Energy and Food, $M_t^J(i)$ can be interpreted as only the continuum of importable goods or the composite between energy and the importable goods, since firm take the quantity of energy as exogenous and so it has been normalized to one.

3. Commodity:

The commodity is assumed to be an exogenous and stochastic endowment, Y_t^{Co} which has its own trend A_t^{Co} that cointegrates with the other sectors, $A_t^{Co} = (A_{t-1}^{Co})^{1-\Gamma_{Co}} (A_t^N)^{\Gamma_{Co}}$. We assume $y_t^{Co} \equiv \frac{Y_t^{Co}}{A_{t-1}^{Co}}$ follows an exogenous process. The endowment is exported at the international price P_t^{Co*} . It is assumed that a fraction ϑ of commodity production is owned by the government and the rest, $(1 - \vartheta)$, is owned by foreigners.

D.1.5 Fiscal and Monetary Policy

The fiscal policy introduces an exogenous expenditure that is completely spent in non-tradable goods. The government receives part of the profits of the commodity sector, can buy local bonds, B_t^G , and gives transfers to households, T_t . Its budget constraint is

$$\vartheta S_t P_t^{Co*} Y_t^{Co} + R_{t-1} B_{t-1}^G = P_t^N G_t + T_t + B_t^G.$$

Similarly to the household, government expenditure is the same composite of non-tradable varieties. We assume $g_t \equiv \frac{G_t}{A_{t-1}^N}$ follows an exogenous process.

Monetary policy follows a Taylor-type rule of the form,

$$\left(\frac{R_t}{R} \right) = \left(\frac{R_{t-1}}{R} \right)^{\varrho_R} \left[\left(\frac{(\pi_t^{NFE})^{\alpha_\pi} \pi_t^{1-\alpha_\pi^{NFE}}}{\bar{\pi}} \right)^{\alpha_\pi} \left(\frac{GDP_t / GDP_{t-1}}{a} \right)^{\alpha_Y} \right]^{1-\varrho_R} e_t^m$$

where the variables without a time subscript are steady state values, π_t^{NFE} is core inflation, GDP_t is gross domestic product and e_t^m is a monetary shock.

D.1.6 Foreign Sector

The rest of the world sells the imported inputs at price $P_{m,t}^*$, buys the commodity at price $P_t^{Co,*}$ and buys the exported products Y_t^X at the price set by local producers. For these last goods, the aggregator of the varieties is the same as for the households. In contrast, the demand for the composite exportable is,

$$C_t^{X,*} = \left(\frac{P_t^X}{S_t P_t^*} \right)^{-\epsilon^*} Y_t^* \xi_t^{X*},$$

where P_t^* is the external CPI index, Y_t^* is external demand and ξ_t^{X*} is a disturbance to external demand; all of them assumed to be exogenous stochastic processes⁴⁶.

The closing device of the model is given by the equation for the international interest rate,

$$R_t^* = R_t^W \exp \left\{ \phi_B \left(\bar{b} - \frac{S_t B_t^*}{P_t^Y GDP_t} \right) \right\} \xi_t^{R1} \xi_t^{R2}. \quad (D.1)$$

In this way, the external rate relevant for the country is composed by three parts. The first part is R_t^W that represents the world interest rate (which in the data is matched with the LIBOR rate). The second part is the term $\exp \left\{ \phi_B \left(\bar{b} - \frac{S_t B_t^*}{P_t^Y GDP_t} \right) \right\} \xi_t^{R1}$, which represents the country premium (equal to the EMBI Chile), where ξ_t^{R1} is an exogenous shock.⁴⁷ Finally, the third part is ξ_t^{R2} , which is a risk-premium shock that captures deviations from the EMBI-adjusted uncovered interest parity (UIP).

D.1.7 Driving Forces

The model features a total of 23 exogenous state variables. Those of domestic origin are consumption preferences (ξ_t^β), labor supply ($\xi_t^{H,N}$ and $\xi_t^{H,X}$), stationary productivity (z_t^H and z_t^X), the growth rate of the long-run trend (a_t), desired markups (ξ_t^N , ξ_t^X and ξ_t^M), endowment of commodities (y_t^{Co}), the relative prices of food and energy (p_t^F and p_t^E), efficiency of investment (u_t), government consumption (g_t), and monetary policy (e_t^m). In turn foreign driving forces are the world interest rate (R_t^W), foreign risk premium (ξ_t^{R1} and ξ_t^{R2}), international prices of commodities (P^{Co*}), imported goods (P_t^{M*}) and CPI for trade partners (P_t^*), demand for exports of X (ξ_t^{X*}), and GDP of trade partners (y_t^*). All these processes are assumed to be Gaussian in logs. Markup and monetary-policy shocks are i.i.d. while the rest, with the exception of international prices, are independent AR(1) processes.

As the model features a balanced growth path and preferences are such that relative prices are stationary, foreign prices should co-integrate, growing all at the same long-run rate.⁴⁸ Defining inflation of foreign CPI as $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$, with steady state value of π^* , we propose the following model for international prices,

$$P_t^j = (\pi^* P_{t-1}^j)^{\Gamma_j} (F_t^*)^{1-\Gamma_j} u_t^j, \quad \text{with } \Gamma_j \in [0, 1), \quad \text{for } j = \{Co*, M*, *\}, \quad (D.2)$$

$$\Delta F_t^* \equiv \frac{F_t^*}{F_{t-1}^*}, \quad \frac{\Delta F_t^*}{\pi^*} = \left(\frac{\Delta F_{t-1}^*}{\pi^*} \right)^{\rho_{F*}} \exp(\epsilon_t^{F*}), \quad \text{with } \rho_{F*} \in (-1, 1) \quad (D.3)$$

$$u_t^j = \left(u_{t-1}^j \right)^{\rho_j} \exp(\epsilon_t^j), \quad \text{with } \rho_j \in (-1, 1), \quad \text{for } j = \{Co*, M*, *\}, \quad (D.4)$$

where ϵ_t^i are i.i.d. $\mathcal{N}(0, \sigma_i^2)$ for $i = \{Co*, M*, *, F*\}$.

⁴⁶We assume foreign inflation, π_t^* , and $y_t^* \equiv \frac{Y_t^*}{A_{t-1}^N}$ follow exogenous processes.

⁴⁷ P_t^Y is the GDP deflator.

⁴⁸In other words, the co-integration vector between the log of any pair of these prices should be $(1, -1)$.

Under this specification, each price is driven by two factors: a common trend (F_t^*) and a price-specific shock (u_t^j). The parameter Γ_j determines how slowly changes in the trend affect each price. The presence of a common trend generates co-integration among prices (as long as $\Gamma_j < 1$), and the fact that the exponent in the trend and in the lagged price in (D.2) add-up to one forces relative prices to remain constant in the long run.⁴⁹ The usual assumption for these prices in DSGE models with nominal rigidities is obtained as a restricted version of this setup, imposing $\Gamma_j = 0$ for $j = \{Co*, M*\}$ and $\sigma_*^2 = 0$. In other words, the relative prices of both commodities and imports are driven by stationary AR(1) processes, while the inflation of commercial partners is a stationary AR(1) process. The specification in (D.2)-(D.4) generalizes this usual assumption in several dimensions. First, in the usual set up, the common trend of all prices is exactly equal to the CPI of commercial partners. This might lead to the wrong interpretation that inflation of commercial partners is a significant driver of domestic variables, while in reality this happens because it represents a common trend in all prices. Second, the usual specification imposes that every change in the common trend has a contemporaneous one-to-one impact in all prices, while in reality different prices may adjust to changes in this common trend at different speeds. Finally, for our specific sample the data favors the general specification (D.2)-(D.4) relative to the restricted model.

Overall, the model features 24 exogenous disturbances, related to the 23 exogenous state variables previously listed plus the common trend in international prices.

D.2 Parametrization Strategy and Goodness of Fit

The values of the parameters in the model are assigned by a combination of calibration and estimation. The resulting values are presented in tables D.2 to D.5. Parameters representing shares in the different aggregate baskets and production functions are calibrated using input-output tables for Chile. In addition, we target several steady-state ratios to sample averages of their observable counterparts. For parameters that are not properly identified in our data set, we rely on studies estimating DSGE models for Chile. Finally, the parameters characterizing the dynamics of some of the external driving forces are calibrated by estimating AR(1) processes.

The remainder of the parameters are estimated with a Bayesian approach using the following series at quarterly frequency from 2001.Q3 to 2016.Q3:⁵⁰

- Real growth rate of: GDP , GDP^X (Agriculture, Fishing, Industry, Utilities, Transportation), GDP^N (Construction, Retail, Services), GDP^{Co} (Mining), private consumption (C), total investment (I), and government consumption (G).
- The ratio of nominal trade balance to GDP.
- Quarterly CPI-based inflation of π^N (services, excluding food and energy), π^T (goods. ex. food and energy), π^M (imported goods, ex. food and energy), π^F (food) and π^E (energy).
- The growth of nominal wages (π^{WX} and π^{WN}) measured as the cost per unit of labor (the CMO index), using sectors consistent with the GDPs definition.
- The nominal dollar exchange-rate depreciation (π^S) and the monetary policy rate (R).
- External: World interest rate (R^W , LIBOR), country premium (EMBI Chile), foreign inflation (π^* , inflation index for commercial partners, the IPE Index), inflation of commodity prices (π^{Co*} ,

⁴⁹If $\Gamma_j = 1$, each price is a random walk with a common drift π^* . Although this implies that in the long run all prices will grow at the same rate, they will not be co-integrated and relative prices may be non-stationary.

⁵⁰The source is the Central Bank of Chile. Variables are seasonally adjusted using the X-11 filter, expressed in logs, multiplied by 100, and demeaned. All growth rates are changes from two consecutive quarters.

copper price) and imports (π^{M*} , price index for imported of goods, the IVUM index), external GDP (Y^* , GDP of commercial partners).

All domestic observables are assumed to have a measurement error, with calibrated variance equal to 10% of the observable variance⁵¹. Priors and posteriors are shown in tables D.3 to D.5. When possible, priors are set centering the distributions around previous results in the literature. The estimated model is able to properly match the volatilities and first-order autocorrelation coefficients of the domestic observables, as can be seen in Table D.1.

Table D.1: Second Moments in the Data and in the Model

Variable	St. Dev. (%)			AC(1)		
	Data		Model	Data		Model
ΔGDP	0.9	0.1	1	0.5	0.2	0.3
$\Delta CONS$	1	0.1	0.8	0.7	0.2	0.7
ΔINV	3.9	0.4	3.8	0.3	0.1	0.6
ΔGDP^X	1.5	0.1	1.5	0.2	0.1	-0.1
ΔGDP^N	0.8	0.1	1.4	0.7	0.1	0.4
TB/GDP	5.5	0.5	4.4	0.8	0.1	0.9
π	0.6	0.1	0.5	0.6	0.2	0.5
π^T	0.7	0.1	0.7	0.6	0.1	0.8
π^M	0.8	0.1	0.7	0.7	0.1	0.7
π^N	0.4	0.0	0.3	0.7	0.2	0.8
π^{WX}	0.6	0.0	0.7	0.6	0.1	0.8
π^{WN}	0.4	0.0	0.3	0.8	0.2	0.9
R	0.4	0.0	0.5	0.9	0.2	0.9
π^S	5.2	0.7	5.1	0.2	0.2	0

Note: The variables are: the growth rates of GDP, private consumption, investment, and GDP in the X and N sectors, the trade-balance-to-output ratio, inflation for total CPI, tradables, non-tradables and imported, the growth rate of nominal wages in sector X and N , the monetary policy rate, and the nominal depreciation. Columns two to four correspond to standard deviations, while five to seven are first-order autocorrelations. For each of these moments, the three columns shown are: point estimates in the data, GMM standard-errors in the data, and unconditional moments in the model evaluated at the posterior mode.

D.2.1 Calibrated and Estimated Parameters

⁵¹Except for the interest rate.

Table D.2: Calibrated

Para.	Descrip.	Value	Source
σ	Risk Aversion	1	Medina and Soto (2007)
φ	Inv. Frish elast.	1	Medina and Soto (2007)
γ	Share C^N in C^{NFE}	0.62	I-O Matrix, average 08-13
γ_T	Share C^X in C^T	0.23	I-O Matrix, average 08-13
γ_I	Share I^N in I	0.62	I-O Matrix, average 08-13
γ_{TI}	Share I^X in I^T	0.02	I-O Matrix, average 08-13
γ_{EC}	Share C^E in C	0.09	I-O Matrix, average 08-13
γ_{FC}	Share C^F in C	0.19	I-O Matrix, average 08-13
α_X	Capital in V.A. X	0.66	I-O Matrix, average 08-13
α_N	Capital in V.A. N	0.49	I-O Matrix, average 08-13
$1 - \gamma_X$	Imports in Prod. X	0.2	I-O Matrix, average 08-13
$1 - \gamma_N$	Imports in Prod. M	0.08	I-O Matrix, average 08-13
γ_{EM}	Share E in Interim. Imports	0.09	I-O Matrix, average 08-13
s^{TB}	Ratio of TB to PIB	0.05	Average 01-15
s^{PIB_N}	Ratio of PIB^N to PIB	0.6	Average 01-15
s^{Co}	Ratio of Co to GDP	0.1	Average 01-15
s^G	Ratio of G to GDP	0.12	Average 01-15
ϑ	Fraction sector Co owned by Gov.	0.56	Average 01-15 ^a
ξ^{R1}	EMBI Chile (annual)	1.015	Average 01-15
π	Inflation (annual)	1.03	Average 01-15
a	Long-run growth (annual)	1.016	Average 01-15
R^W	World Interest Rate (annual)	1.045	Average 01-15
R	Monetary Policy Rate (annual)	1.058	Average 01-15
ϕ_B	Elasticity R^* to debt	0.005	Av. value for Chile ^b
δ	Capital depreciation	0.01	Medina and Soto (2007)
ϵ^j	Elast. of Subst. Varieties	11	Medina and Soto (2007)

Notes: ^a This includes the public production and the taxes received by the government of the rest of the production. ^b See for example recent DSGE's in [Kirchner and Tranamil \(2016\)](#) and [García-Cicco et al. \(2015\)](#).

Table D.3: Estimated Parameters

Para.	Description	Dist.	Prior		Posterior	
			Mean	St.D.	Mode	St.D.
ϕ_C	Habits C	β	0.65	0.2	0.879	0.03
ϕ_I	Inv. Adj. Costs	\mathcal{N}^+	4	1	4.461	0.74
θ_{WX}	Calvo W^X	β	0.65	0.2	0.940	0.01
ζ_{WX}	Din. Index. W^X	β	0.5	0.27	0.066	0.11
θ_{WN}	Calvo W^N	β	0.65	0.2	0.969	0.01
ζ_{WN}	Din. Index. W^N	β	0.5	0.27	0.117	0.08
ϱ	Sust. C^T, C^N	\mathcal{N}^+	0.9	1.5	0.171	0.85
ϱ_I	Sust. I^T, I^N	\mathcal{N}^+	0.9	1.5	2.339	1.12
θ_X	Calvo X	β	0.5	0.27	0.600	0.07
θ_M	Calvo M	β	0.5	0.27	0.858	0.02
θ_N	Calvo N	β	0.5	0.27	0.952	0.01
ϱ_X	Index. Own X	β	0.5	0.27	0.887	0.22
ϱ_M	Index. Own M	β	0.5	0.27	0.662	0.28
ϱ_N	Index. Own N	β	0.5	0.27	0.775	0.11
ζ_X	Din. Index. X	β	0.5	0.27	0.919	0.17
ζ_M	Din. Index. M	β	0.5	0.27	0.541	0.18
ζ_N	Din. Index. N	β	0.5	0.27	0.817	0.10
Γ_X	Adj. Trend X	β	0.65	0.2	0.772	0.25
Γ_{Co}	Adj. Trend Co	β	0.65	0.2	0.763	0.25
Policy Rule						
ρ_R	Smoothing	β	0.8	0.05	0.786	0.03
α_π	Reaction to π	\mathcal{N}^+	1.7	0.1	1.630	0.09
α_π^{SAE}	Reaction to π^{NFE}	β	0.5	0.2	0.439	0.18
α_y	Reaction to y	\mathcal{N}^+	0.125	0.05	0.145	0.05
ϵ^*	Elast. Ext. Dem.	IG	0.3	0.1	0.198	0.04

Note: Prior distributions: β Beta, \mathcal{N}^+ Normal truncated for positive values, IG Inverse Gamma, \mathcal{U} Uniform. The standard deviation of the posterior is approximated by the inverse Hessian evaluated at the posterior mode.

Table D.4: Estimated Parameters, Coefficients Dynamics of Exogenous Processes

Para.	Dist.	Prior		Posterior	
		Mean	St.D.	Mode	St.D.
Dynamics of Driving Forces					
$\rho_{\xi\beta}$	β	0.65	0.2	0.777	0.08
ρ_a	β	0.35	0.15	0.286	0.16
ρ_u	β	0.65	0.2	0.545	0.11
ρ_{z^X}	β	0.5	0.23	0.910	0.06
ρ_{z^N}	β	0.5	0.23	0.693	0.10
$\rho_{\xi^{X*}}$	β	0.65	0.2	0.871	0.05
$\rho_{\xi^{R1}}$	β	0.65	0.2	0.946	0.02
$\rho_{\xi^{R2}}$	β	0.65	0.2	0.734	0.12
$\rho_{\xi^{hX}}$	β	0.65	0.2	0.829	0.08
$\rho_{\xi^{hN}}$	β	0.65	0.2	0.919	0.05
ρ_{p^A}	β	0.65	0.2	0.973	0.02
ρ_{p^E}	β	0.65	0.2	0.895	0.05
Γ_*	\mathcal{U}	0.5	0.29	0.304	0.12
Γ_{M*}	\mathcal{U}	0.5	0.29	0.488	0.08
Γ_{Co*}	\mathcal{U}	0.5	0.29	0.161	0.12
ρ_{F*}	\mathcal{U}	0	0.58	0.206	0.11
ρ_*	\mathcal{U}	0	0.58	0.737	0.15
ρ_{M*}	\mathcal{U}	0	0.58	0.561	0.10
ρ_{Co*}	\mathcal{U}	0	0.58	0.892	0.05
ρ_{yCo}	β	0.55	0.2	0.881	0.06

Table D.5: Estimated Parameters, Standard Deviations Exogenous shocks

Para.	Dist.	Prior		Posterior	
		Mean	St.D.	Mode	St.D.
σ^{ξ^B}	\mathcal{N}^+	0.03	0.03	0.042	0.011
σ^a	IG	0.01	∞	0.004	0.001
σ^u	\mathcal{N}^+	0.03	0.03	0.078	0.017
σ^{z^X}	\mathcal{N}^+	0.01	0.03	0.009	0.003
σ^{z^N}	\mathcal{N}^+	0.005	0.03	0.044	0.006
σ^{ξ^X}	\mathcal{N}^+	0.1	0.3	0.006	0.064
σ^{ξ^M}	\mathcal{N}^+	0.1	0.3	0.163	0.047
σ^{ξ^N}	\mathcal{N}^+	0.1	0.3	0.726	0.184
σ^{e^m}	\mathcal{N}^+	0.01	0.01	0.002	0.000
$\sigma^{\xi^{X*}}$	\mathcal{N}^+	0.01	0.01	0.021	0.002
$\sigma^{\xi^{R1}}$	\mathcal{N}^+	0.01	0.01	0.001	0.000
$\sigma^{\xi^{R2}}$	\mathcal{N}^+	0.01	0.01	0.004	0.003
$\sigma^{\xi^{hN}}$	\mathcal{N}^+	0.1	0.15	0.244	0.103
$\sigma^{\xi^{hN}}$	\mathcal{N}^+	0.1	0.15	0.165	0.089
σ^{p^A}	\mathcal{N}^+	0.02	0.02	0.011	0.001
σ^{p^E}	\mathcal{N}^+	0.04	0.04	0.025	0.003
σ^{F*}	\mathcal{U}	0.25	0.14	0.028	0.005
σ^*	\mathcal{U}	0.25	0.14	0.014	0.002
σ^{M*}	\mathcal{U}	0.25	0.14	0.014	0.002
σ^{Co*}	\mathcal{U}	0.25	0.14	0.120	0.011
$\sigma^{y^{Co}}$	\mathcal{N}^+	0.02	0.02	0.022	0.002

D.3 Optimality Conditions

D.3.1 Household

From the decision of final consumption, labor, bonds and capital and defining as λ_t the multiplier of the budget constraint, $\mu_t^J \lambda_t$ the multiplier of the capital accumulation equation for $J = \{X, N\}$ and as $\mu_t^{WJ} W_t^J \lambda_t$ the multiplier of the equalization of labor demand and supply, we have the first order conditions:

$$\begin{aligned}
& \xi_t^\beta (C_t - \phi_c \tilde{C}_{t-1})^{-\sigma} - P_t \lambda_t = 0 \\
& -\xi_t^\beta \kappa_t \xi_t^{h,J} (h_t^J)^\varphi + \mu_t^{WJ} W_t^J \lambda_t = 0 \\
& -\lambda_t + \beta E_t \lambda_{t+1} R_t = 0 \\
& -\lambda_t S_t + \beta E_t \lambda_{t+1} S_{t+1} R_t^* = 0 \\
& -\mu_t^J \lambda_t + \beta E_t \{ \lambda_{t+1} P_{t+1}^J R_{t+1}^J + \mu_{t+1}^J \lambda_{t+1} (1 - \delta) \} = 0 \\
& -\lambda_t P_t^I + \mu_t^J \lambda_t \left\{ \left[1 - \Gamma \left(\frac{I_t^J}{I_{t-1}^J} \right) \right] u_t + \left(-\Gamma' \left(\frac{I_t^J}{I_{t-1}^J} \right) \frac{1}{I_{t-1}^J} \right) u_t I_t^J \right\} + \\
& \beta E_t \left\{ \mu_{t+1}^J \lambda_{t+1} \left(-\Gamma' \left(\frac{I_{t+1}^J}{I_t^J} \right) \right) \left(-\frac{I_{t+1}^J}{(I_t^J)^2} \right) u_{t+1} I_{t+1}^J \right\} = 0
\end{aligned}$$

where $J = \{X, N\}$ for the second and last two equations. The functional form for $\Gamma(x)$ is:

$$\Gamma(x) = 1 - \frac{\phi_I}{2} (x - a)^2$$

where a is the steady state value of the trend growth. From the optimality conditions of choosing wages, we can write the first order conditions as:

$$\begin{aligned}
& \frac{\epsilon_W - 1}{\epsilon_W} W_t^{J,*} E_t \sum_{\tau=0}^{\infty} (\theta_{WJ} \beta)^\tau \lambda_{t+\tau} \left\{ \frac{h_{t+\tau}^{J,d}}{(W_{t+\tau}^J)^{-\epsilon_W}} (W_t^{J,*})^{-\epsilon_W} \left[a^\tau \prod_{s=1}^{\tau} ((\pi_{t+s-1}^J)^{\varrho_{WJ}} \pi_{t+s-1}^{1-\varrho_{WJ}})^{\zeta_{WJ}} \bar{\pi}_{t+s}^{1-\zeta_{WJ}} \right]^{1-\epsilon_W} \right\} = \\
& E_t \sum_{\tau=0}^{\infty} (\theta_{WJ} \beta)^\tau \mu_{t+\tau}^{WJ} \lambda_{t+\tau} W_{t+\tau}^J \left\{ \frac{h_{t+\tau}^{J,d}}{(W_{t+\tau}^J)^{-\epsilon_W}} (W_t^{J,*})^{-\epsilon_W} \left[a^\tau \prod_{s=1}^{\tau} ((\pi_{t+s-1}^J)^{\varrho_{WJ}} \pi_{t+s-1}^{1-\varrho_{WJ}})^{\zeta_{WJ}} \bar{\pi}_{t+s}^{1-\zeta_{WJ}} \right]^{-\epsilon_W} \right\}
\end{aligned}$$

where $W_t^{J,*}$ is the optimal wage chosen and this equation holds for $J = \{X, N\}$.

In addition, the optimality conditions for the decision between tradable and non-tradable consumption are:

$$\begin{aligned}
C_t^N &= \gamma \left(\frac{P_t^N}{P_t} \right)^{-\varrho} C_t \\
C_t^T &= (1 - \gamma) \left(\frac{P_t^T}{P_t} \right)^{-\varrho} C_t
\end{aligned}$$

where it was used the fact that $C_t^{SAE} = C_t$.

And between the exportable and importable:

$$\begin{aligned} C_t^X &= \gamma_T \left(\frac{P_t^T C_t^T}{P_t^X} \right) \\ C_t^M &= (1 - \gamma_T) \left(\frac{P_t^T C_t^T}{P_t^M} \right) \end{aligned}$$

D.3.2 Investment Good Production

The first order conditions between tradable and non-tradable investment can be written as:

$$\begin{aligned} \tilde{I}_t^N &= \gamma_I \left(\frac{P_t^N}{P_t^I} \right)^{-\varrho_I} I_t \\ \tilde{I}_t^T &= (1 - \gamma_I) \left(\frac{P_t^{TI}}{P_t^I} \right)^{-\varrho_I} I_t \end{aligned}$$

where P_t^{TI} is the price index defined for the tradable investment. The FOC between exportable and importable investment is given by:

$$\begin{aligned} \tilde{I}_t^X &= \gamma_{TI} \left(\frac{P_t^{TI} \tilde{I}_t^T}{P_t^X} \right) \\ \tilde{I}_t^M &= (1 - \gamma_{TI}) \left(\frac{P_t^{TI} \tilde{I}_t^T}{P_t^M} \right) \end{aligned}$$

D.3.3 Firms

The first order conditions are the same for each firm i in each sector and so the subscript will be omitted. First, given the marginal costs, the first order condition of the price setting can be written as:

$$\begin{aligned} \xi_t^J \frac{\epsilon_J - 1}{\epsilon_J} (P_t^{J,*})^{-\epsilon_J} \sum_{\tau=0}^{\infty} (\beta \theta_J)^\tau \Lambda_{t,t+\tau} \frac{1}{(P_{t+\tau}^J)^{-\epsilon_J}} Y_{t+\tau}^J \left[\prod_{s=1}^{\tau} \left((\pi_{t+s-1}^J)^{\varrho_J} \pi_{t+s-1}^{1-\varrho_J} \right)^{\zeta_J} \bar{\pi}_{t+s}^{1-\zeta_J} \right]^{1-\epsilon_J} = \\ (P_t^{J,*})^{-\epsilon_J-1} \sum_{\tau=1}^{\infty} (\beta \theta_J)^\tau \Lambda_{t,t+\tau} MC_{t+\tau}^J \frac{1}{(P_{t+\tau}^J)^{-\epsilon_J}} Y_{t+\tau}^J \left[\prod_{s=1}^{\tau} \left((\pi_{t+s-1}^J)^{\varrho_J} \pi_{t+s-1}^{1-\varrho_J} \right)^{\zeta_J} \bar{\pi}_{t+s}^{1-\zeta_J} \right]^{-\epsilon_J} \end{aligned}$$

where $P_t^{J,*}$ is the optimal price chosen at t . To get the marginal cost of each sector, we distinguish between the importable and the other sectors

- Sector M : Cost minimization implies that their marginal cost is the same for all firms and is given by the price in local currency of the imported input:

$$MC_t^M = P_{m,t}$$

Note the difference between the price set by the M sector, P_t^M , and the price of its input, $P_{m,t}$.

- Sector X and N :

1. Optimal production of V_t^J : The optimality conditions and the marginal cost are:

$$\begin{aligned} h_t^{J,d} &= \frac{V_t^J}{z_t^J (A_t^J)^{1-\alpha_J}} \left[\frac{1-\alpha_J}{\alpha_J} \frac{P_t^J R_t^J}{W_t^J} \right]^{\alpha_J} \\ K_{t-1}^J &= \frac{V_t^J}{z_t^J (A_t^J)^{1-\alpha_J}} \left[\frac{\alpha_J}{1-\alpha_J} \frac{W_t^J}{P_t^J R_t^J} \right]^{1-\alpha_J} \\ MC_t^{V,J} &= \frac{1}{z_t^J (A_t^J)^{1-\alpha_J}} (P_t^J R_t^J)^{\alpha_J} (W_t^J)^{1-\alpha_J} \left[\frac{1}{(1-\alpha_J)^{1-\alpha_J} \alpha_J^{\alpha_J}} \right] \end{aligned}$$

2. Optimal production Y_t^J :

$$\begin{aligned} M_t^J &= Y_t^J(i) \left[\frac{1-\gamma_J}{\gamma_J} \frac{MC_t^{V,J}}{P_t^{ME}} \right]^{\gamma_J} \\ V_t^J &= Y_t^J(i) \left[\frac{\gamma_J}{1-\gamma_J} \frac{P_t^{ME}}{MC_t^{V,J}} \right]^{1-\gamma_J} \\ MC_t^J &= (MC_t^{V,J})^{\gamma_J} (P_t^{ME})^{1-\gamma_J} \left[\frac{1}{(1-\gamma_J)^{1-\gamma_J} \gamma_J^{\gamma_J}} \right] \end{aligned}$$

where MC_t^J is the marginal cost of producing Y_t^J .

D.3.4 Market Clearing

All markets clear:

$$\begin{aligned} B_t &= B_t^G \\ I_t &= I_t^X + I_t^N \\ h_t^X &= \Delta_t^{WX} h_t^{X,d} \\ h_t^N &= \Delta_t^{WN} h_t^{N,d} \\ Y_t^X &= \Delta_t^X \left(C_t^X + \tilde{I}_t^X + C_t^{X,*} \right) \\ Y_t^M &= \Delta_t^M \left(C_t^M + \tilde{I}_t^M + M_t^X + M_t^N \right) \\ Y_t^N &= \Delta_t^N \left(C_t^N + \tilde{I}_t^N + G_t^N \right) \end{aligned}$$

Which correspond to the local bonds market, the investment market, labor markets and goods market. The Δ variables are measures of dispersion in prices in the different markets, given by:

$$\begin{aligned} \Delta_t^{WJ} &= \int_0^1 \left(\frac{W_t^J(i)}{W_t^J} \right)^{-\epsilon_W} dj \\ \Delta_t^J &= \int_0^1 \left(\frac{P_t^J(i)}{P_t^J} \right)^{-\epsilon_J} dj \end{aligned}$$

the first equation for $J = \{X, N\}$ and the second for $J = \{X, M, N\}$. The rest of the equations correspond to the policy and foreign equations described in the previous section.

D.4 Equilibrium Conditions

This sections describes the equilibrium conditions after the variables were redefined to make them stationary. The transformations made to the variables were: all lower case prices are the corresponding capital price divided by the CPI Index with the exception of $p_t^{Co,*}$ and $p_t^{M,*}$ which are divided by the foreign CPI price index and $p_t^{J,*} = P_t^{J,*}/P_t^J$ for $J = \{X, M, N\}$. All lower case real variables (consumption, investment, capital, government expenditure, production, imports, productivity, output, foreign demand) are the upper case divided by A_{t-1} with the exception of $y_t^{Co} = Y_t^{Co}/A_{t-1}^{Co}$. All inflation definitions are the corresponding price index divided by the price index in the previous period. And particular definitions are: $\tilde{\xi}_t^{h,J} = \xi_t^{h,J}/A_{t-1}$, $\tilde{\mu}_t^J = \mu_t^J/P_t$, $b_t^* = B_t^*/(A_{t-1}P_t^*)$, $\tilde{f}_t^J = f_t^{1,J}/(A_{t-1}P_t^\sigma)$, $\tilde{f}_t^{WJ} = f_t^{WJ}/A_{t-1}^{1-\sigma}$, $\tilde{\lambda}_t = P_t\lambda_t/A_{t-1}^{-\sigma}$, $w_t^J = W_t^J/(A_{t-1}P_t)$, $w_t^{J,*} = W_t^{J,*}/W_t^J$, $mc_t^J = MC_t^J/P_t^J$ and $mc_t^{V,J} = MC_t^{V,J}/P_t^J$ for $J = \{X, M, N\}$ or $J = \{X, N\}$ depending on the variable. In addition, new variables were defined as the real exchange rate, the trade balance, the gdp deflator among others.

There are 80 endogenous variables,

$$\{c_t, \tilde{\lambda}_t, h_t^X, \mu_t^{WX}, w_t^X, h_t^N, \mu_t^{WN}, w_t^N, R_t, \pi_t, R_t^*, \pi_t^S, \tilde{\mu}_t^X, p_t^X, R_t^X, \tilde{\mu}_t^N, p_t^N, R_t^N, p_t^I, i_t^X, i_t^N, k_t^X, k_t^N, \tilde{f}_t^{WX}, w_t^{X,*}, h_t^{X,d}, \pi_t^X, \tilde{f}_t^{WN}, w_t^{N,*}, h_t^{N,d}, \pi_t^N, c_t^N, p_t^{SAE}, c_t^T, p_t^T, c_t^X, p_t^M, c_t^M, p_t^{TI}, \tilde{i}_t^N, \tilde{i}_t^T, \tilde{i}_t^X, \tilde{i}_t^M, i_t, mc_t^M, y_t^M, m_t, p_{m,t}, v_t^X, a_t^X, v_t^N, mc_t^{V,X}, mc_t^{V,N}, y_t^X, p_t^{ME}, m_t^X, y_t^N, m_t^N, mc_t^X, mc_t^N, \tilde{f}_t^X, p_t^{X,*}, \tilde{f}_t^M, p_t^{M,*}, \pi_t^M, \tilde{f}_t^N, p_t^{N,*}, gdp_t, \pi_t^{SAE}, c_t^{X,*}, rer_t, \Delta_t^{WX}, \Delta_t^{WN}, \Delta_t^N, \Delta_t^X, \Delta_t^M, tb_t, a_t^{Co}, b_t^*, p_t^Y\}$$

and 23 shocks:

$$\{\xi_t^\beta, a_t, \tilde{\xi}_t^{h,X}, \tilde{\xi}_t^{h,N}, \xi_t^X, \xi_t^M, \xi_t^N, p_t^A/p_t^T, p_t^E/p_t^T, u_t, z_t^X, z_t^N, g_t, e_t^m, y_t^*, \xi_t^{X,*}, \Delta F_t^*, u_t^*, u_t^{M,*}, u_t^{Co,*}, R_t^W, \xi_t^{R1}, \xi_t^{R2}, y_t^{Co}\}.$$

$$\xi_t^\beta \left(c_t - \phi_C \frac{c_{t-1}}{a_{t-1}} \right)^{-\sigma} = \tilde{\lambda}_t \quad (D.5)$$

$$\tilde{\xi}_t^{h,X} (h_t^X)^\varphi = \mu_t^{WX} w_t^X \quad (D.6)$$

$$\tilde{\xi}_t^{h,N} (h_t^N)^\varphi = \mu_t^{WN} w_t^N \quad (D.7)$$

$$\tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \frac{\tilde{\lambda}_{t+1} R_t}{\pi_{t+1}} \quad (D.8)$$

$$\tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \frac{\tilde{\lambda}_{t+1} R_t^* \pi_{t+1}^S}{\pi_{t+1}} \quad (D.9)$$

$$\tilde{\mu}_t^X \tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \left\{ \tilde{\lambda}_{t+1} p_{t+1}^X R_{t+1}^X + \tilde{\mu}_{t+1}^X \tilde{\lambda}_{t+1} (1 - \delta) \right\} \quad (D.10)$$

$$\tilde{\mu}_t^N \tilde{\lambda}_t = \beta a_t^{-\sigma} E_t \left\{ \tilde{\lambda}_{t+1} p_{t+1}^N R_{t+1}^N + \tilde{\mu}_{t+1}^N \tilde{\lambda}_{t+1} (1 - \delta) \right\} \quad (D.11)$$

$$\begin{aligned}\tilde{\lambda}_t p_t^I = & \tilde{\mu}_t^X \tilde{\lambda}_t \left\{ 1 - \frac{\phi_I}{2} \left(\frac{i_t^X}{i_{t-1}^X} a_{t-1} - a \right)^2 - \phi_I \left(\frac{i_t^X}{i_{t-1}^X} a_{t-1} - a \right) \frac{i_t^X}{i_{t-1}^X} a_{t-1} \right\} u_t + \\ & \beta a_t^{-\sigma} E_t \tilde{\mu}_{t+1}^X \tilde{\lambda}_{t+1} \phi_I \left(\frac{i_{t+1}^X}{i_t^X} a_t - a \right) \left(\frac{i_{t+1}^X}{i_t^X} a_t \right)^2 u_{t+1}\end{aligned}\quad (\text{D.12})$$

$$\begin{aligned}\tilde{\lambda}_t p_t^I = & \tilde{\mu}_t^N \tilde{\lambda}_t \left\{ 1 - \frac{\phi_I}{2} \left(\frac{i_t^N}{i_{t-1}^N} a_{t-1} - a \right)^2 - \phi_I \left(\frac{i_t^N}{i_{t-1}^N} a_{t-1} - a \right) \frac{i_t^N}{i_{t-1}^N} a_{t-1} \right\} u_t + \\ & \beta a_t^{-\sigma} E_t \tilde{\mu}_{t+1}^N \tilde{\lambda}_{t+1} \phi_I \left(\frac{i_{t+1}^N}{i_t^N} a_t - a \right) \left(\frac{i_{t+1}^N}{i_t^N} a_t \right)^2 u_{t+1}\end{aligned}\quad (\text{D.13})$$

$$k_t^X = \left[1 - \frac{\phi_I}{2} \left(\frac{i_t^X}{i_{t-1}^X} a_{t-1} - a \right)^2 \right] u_t i_t^X + (1 - \delta) \frac{k_{t-1}^X}{a_{t-1}} \quad (\text{D.14})$$

$$k_t^N = \left[1 - \frac{\phi_I}{2} \left(\frac{i_t^N}{i_{t-1}^N} a_{t-1} - a \right)^2 \right] u_t i_t^N + (1 - \delta) \frac{k_{t-1}^N}{a_{t-1}} \quad (\text{D.15})$$

$$\begin{aligned}\tilde{f}_t^{WX} = & \frac{\epsilon_W - 1}{\epsilon_W} (w_t^{X,*})^{1-\epsilon_W} \tilde{\lambda}_t h_t^{X,d} + \\ & \theta_{WX} a_t^{1-\sigma} \beta E_t \left(\frac{w_t^{X,*}}{w_{t+1}^{X,*}} \frac{w_t^X}{w_{t+1}^X} \right)^{1-\epsilon_W} \left[\frac{a}{a_t} \frac{((\pi_t^X)^{\varrho_{WX}} \pi_t^{1-\varrho_{WX}}) \zeta_{WX} \bar{\pi}^{1-\zeta_{WX}}}{\pi_{t+1}} \right]^{1-\epsilon_W} \frac{w_{t+1}^X}{w_t^X} \tilde{f}_{t+1}^{WX}\end{aligned}\quad (\text{D.16})$$

$$\begin{aligned}\tilde{f}_t^{WN} = & \frac{\epsilon_W - 1}{\epsilon_W} (w_t^{N,*})^{1-\epsilon_W} \tilde{\lambda}_t h_t^{N,d} + \\ & \theta_{WN} a_t^{1-\sigma} \beta E_t \left(\frac{w_t^{N,*}}{w_{t+1}^{N,*}} \frac{w_t^N}{w_{t+1}^N} \right)^{1-\epsilon_W} \left[\frac{a}{a_t} \frac{((\pi_t^N)^{\varrho_{WN}} \pi_t^{1-\varrho_{WN}}) \zeta_{WN} \bar{\pi}^{1-\zeta_{WN}}}{\pi_{t+1}} \right]^{1-\epsilon_W} \frac{w_{t+1}^N}{w_t^N} \tilde{f}_{t+1}^{WN}\end{aligned}\quad (\text{D.17})$$

$$\begin{aligned}\tilde{f}_t^{WX} = & (w_t^{X,*})^{-\epsilon_W} \mu_t^{WX} \tilde{\lambda}_t h_t^{X,d} + \\ & \theta_{WX} a_t^{1-\sigma} \beta E_t \left(\frac{w_t^{X,*}}{w_{t+1}^{X,*}} \frac{w_t^X}{w_{t+1}^X} \right)^{-\epsilon_W} \left[\frac{a}{a_t} \frac{((\pi_t^X)^{\varrho_{WX}} \pi_t^{1-\varrho_{WX}}) \zeta_{WX} \bar{\pi}^{1-\zeta_{WX}}}{\pi_{t+1}} \right]^{-\epsilon_W} \frac{w_{t+1}^X}{w_t^X} \tilde{f}_{t+1}^{WX}\end{aligned}\quad (\text{D.18})$$

$$\begin{aligned}\tilde{f}_t^{WN} = & (w_t^{N,*})^{-\epsilon_W} \mu_t^{WN} \tilde{\lambda}_t h_t^{N,d} + \\ & \theta_{WN} a_t^{1-\sigma} \beta E_t \left(\frac{w_t^{N,*}}{w_{t+1}^{N,*}} \frac{w_t^N}{w_{t+1}^N} \right)^{-\epsilon_W} \left[\frac{a}{a_t} \frac{((\pi_t^N)^{\varrho_{WN}} \pi_t^{1-\varrho_{WN}}) \zeta_{WN} \bar{\pi}^{1-\zeta_{WN}}}{\pi_{t+1}} \right]^{-\epsilon_W} \frac{w_{t+1}^N}{w_t^N} \tilde{f}_{t+1}^{WN}\end{aligned}\quad (\text{D.19})$$

$$1 = \theta_{WX} \left(\frac{w_{t-1}^X}{w_t^X} \frac{a}{a_{t-1}} \frac{((\pi_{t-1}^X)^{\varrho_{WX}} \pi_{t-1}^{1-\varrho_{WX}}) \zeta_{WX} \bar{\pi}^{1-\zeta_{WX}}}{\pi_t} \right)^{1-\epsilon_W} + (1 - \theta_{WX}) (w_t^{X,*})^{1-\epsilon_W} \quad (\text{D.20})$$

$$1 = \theta_{WN} \left(\frac{w_{t-1}^N}{w_t^N} \frac{a}{a_{t-1}} \frac{((\pi_{t-1}^N)^{\varrho_{WN}} \pi_{t-1}^{1-\varrho_{WN}})^{\zeta_{WN}} \bar{\pi}^{1-\zeta_{WN}}}{\pi_t} \right)^{1-\epsilon_W} + (1 - \theta_{WN}) \left(w_t^{N,*} \right)^{1-\epsilon_W} \quad (\text{D.21})$$

$$c_t^N = \gamma \left(\frac{p_t^N}{p_t^{SAE}} \right)^{-\varrho} c_t \quad (\text{D.22})$$

$$c_t^T = (1 - \gamma) \left(\frac{p_t^T}{p_t^{SAE}} \right)^{-\varrho} c_t \quad (\text{D.23})$$

$$c_t^X = \gamma_T \left(\frac{p_t^T c_t^T}{p_t^X} \right) \quad (\text{D.24})$$

$$c_t^M = (1 - \gamma_T) \left(\frac{p_t^T c_t^T}{p_t^M} \right) \quad (\text{D.25})$$

$$1 = (p_t^{SAE})^{1-\gamma_{AC}-\gamma_{EC}} (p_t^A)^{\gamma_{AC}} (p_t^E)^{\gamma_{EC}} \quad (\text{D.26})$$

$$1 = (1 - \gamma) \left(\frac{p_t^T}{p_t^{SAE}} \right)^{1-\varrho} + \gamma \left(\frac{p_t^N}{p_t^{SAE}} \right)^{1-\varrho} \quad (\text{D.27})$$

$$p_t^T = (p_t^X)^{\gamma_T} (p_t^M)^{1-\gamma_T} \quad (\text{D.28})$$

$$p_t^I = (\gamma_I (p_t^N)^{1-\varrho_I} + (1 - \gamma_I) (p_t^{TI})^{1-\varrho_I})^{\frac{1}{1-\varrho_I}} \quad (\text{D.29})$$

$$p_t^{TI} = (p_t^X)^{\gamma_{TI}} (p_t^M)^{1-\gamma_{TI}} \quad (\text{D.30})$$

$$\tilde{i}_t^N = \gamma_I \left(\frac{p_t^N}{p_t^I} \right)^{-\varrho_I} i_t \quad (\text{D.31})$$

$$\tilde{i}_t^T = (1 - \gamma_I) \left(\frac{p_t^{TI}}{p_t^I} \right)^{-\varrho_I} i_t \quad (\text{D.32})$$

$$\tilde{i}_t^X = \gamma_{TI} \left(\frac{p_t^{TI} \tilde{i}_t^T}{p_t^X} \right) \quad (\text{D.33})$$

$$\tilde{i}_t^M = (1 - \gamma_{TI}) \left(\frac{p_t^{TI} \tilde{i}_t^T}{p_t^M} \right) \quad (\text{D.34})$$

$$m c_t^M = \frac{p_{m,t}}{p_t^M} \quad (\text{D.35})$$

$$y_t^M = m_t \quad (\text{D.36})$$

$$h_t^{X,d} = \frac{v_t^X}{z_t^X (a_t^X)^{1-\alpha_X}} \left[\frac{1 - \alpha_X}{\alpha_X} \frac{p_t^X}{w_t^X} R_t^X \right]^{\alpha_X} \quad (\text{D.37})$$

$$k_{t-1}^X = a_{t-1} \frac{v_t^X}{z_t^X (a_t^X)^{1-\alpha_X}} \left[\frac{\alpha_X}{1-\alpha_X} \frac{w_t^X}{p_t^X R_t^X} \right]^{1-\alpha_X} \quad (\text{D.38})$$

$$h_t^{N,d} = \frac{v_t^N}{z_t^N a_t^{1-\alpha_N}} \left[\frac{1-\alpha_N}{\alpha_N} \frac{p_t^N}{w_t^N} R_t^N \right]^{\alpha_N} \quad (\text{D.39})$$

$$k_{t-1}^N = a_{t-1} \frac{v_t^N}{z_t^N a_t^{1-\alpha_N}} \left[\frac{\alpha_N}{1-\alpha_N} \frac{w_t^N}{p_t^N R_t^N} \right]^{1-\alpha_N} \quad (\text{D.40})$$

$$mc_t^{V,X} = \frac{1}{z_t^X (a_t^X)^{1-\alpha_X}} \frac{(p_t^X R_t^X)^{\alpha_X} (w_t^X)^{1-\alpha_X}}{p_t^X} \left[\frac{1}{(1-\alpha_X)^{1-\alpha_X} \alpha_X^{\alpha_X}} \right] \quad (\text{D.41})$$

$$mc_t^{V,N} = \frac{1}{z_t^N a_t^{1-\alpha_N}} \frac{(p_t^N R_t^N)^{\alpha_N} (w_t^N)^{1-\alpha_N}}{p_t^N} \left[\frac{1}{(1-\alpha_N)^{1-\alpha_N} \alpha_N^{\alpha_N}} \right] \quad (\text{D.42})$$

$$v_t^X = y_t^X \left[\frac{\gamma_X}{1-\gamma_X} \frac{p_t^{ME}}{mc_t^{V,X}} \frac{1}{p_t^X} \right]^{1-\gamma_X} \quad (\text{D.43})$$

$$m_t^X = y_t^X \left[\frac{1-\gamma_X}{\gamma_X} \frac{mc_t^{V,X}}{p_t^{ME}} p_t^X \right]^{\gamma_X} \quad (\text{D.44})$$

$$v_t^N = y_t^N \left[\frac{\gamma_N}{1-\gamma_N} \frac{p_t^{ME}}{mc_t^{V,N}} \frac{1}{p_t^N} \right]^{1-\gamma_N} \quad (\text{D.45})$$

$$m_t^N = y_t^N \left[\frac{1-\gamma_N}{\gamma_N} \frac{mc_t^{V,N}}{p_t^{ME}} p_t^N \right]^{\gamma_N} \quad (\text{D.46})$$

$$mc_t^X = (mc_t^{V,X})^{\gamma_X} \left(\frac{p_t^{ME}}{p_t^X} \right)^{1-\gamma_X} \frac{1}{(1-\gamma_X)^{1-\gamma_X} \gamma_X^{\gamma_X}} \quad (\text{D.47})$$

$$mc_t^N = (mc_t^{V,N})^{\gamma_N} \left(\frac{p_t^{ME}}{p_t^N} \right)^{1-\gamma_N} \frac{1}{(1-\gamma_N)^{1-\gamma_N} \gamma_N^{\gamma_N}} \quad (\text{D.48})$$

$$a_t^X = \left(\frac{a_{t-1}^X}{a_{t-1}} \right)^{1-\Gamma_X} (a_t)^{\Gamma_X} \quad (\text{D.49})$$

$$p_t^{ME} = (p_t^M)^{1-\gamma_{EF}} (p_t^E)^{\gamma_{EF}} \quad (\text{D.50})$$

$$\begin{aligned} \tilde{f}_t^X = & \xi_t^X \frac{\epsilon_X - 1}{\epsilon_X} \left(p_t^{X,*} \right)^{1-\epsilon_X} y_t^X + \\ & \beta a_t^{1-\sigma} \theta_X E_t \left(\frac{p_t^{X,*}}{p_{t+1}^{X,*}} \frac{p_t^X}{p_{t+1}^X} \right)^{1-\epsilon_X} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\left((\pi_t^X)^{\varrho_X} \pi_t^{1-\varrho_X} \right)^{\zeta_X} \bar{\pi}^{1-\zeta_X} \right]^{1-\epsilon_X}}{\pi_{t+1}^{1-\epsilon_X}} \frac{\pi_{t+1}^X}{\pi_{t+1}} \tilde{f}_{t+1}^X \end{aligned} \quad (\text{D.51})$$

$$\begin{aligned}\tilde{f}_t^M &= \xi_t^M \frac{\epsilon_M - 1}{\epsilon_M} \left(p_t^{M,*} \right)^{1-\epsilon_M} y_t^M + \\ &\quad \beta a_t^{1-\sigma} \theta_M E_t \left(\frac{p_t^{M,*}}{p_{t+1}^{M,*}} \frac{p_t^M}{p_{t+1}^M} \right)^{1-\epsilon_M} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\left((\pi_t^M)^{\varrho_M} \pi_t^{1-\varrho_M} \right)^{\zeta_M} \bar{\pi}^{1-\zeta_M} \right]^{1-\epsilon_M}}{\pi_{t+1}^{1-\epsilon_M}} \frac{\pi_{t+1}^M}{\pi_{t+1}} \tilde{f}_{t+1}^M\end{aligned}\tag{D.52}$$

$$\begin{aligned}\tilde{f}_t^N &= \xi_t^N \frac{\epsilon_N - 1}{\epsilon_N} \left(p_t^{N,*} \right)^{1-\epsilon_N} y_t^N + \\ &\quad \beta a_t^{1-\sigma} \theta_N E_t \left(\frac{p_t^{N,*}}{p_{t+1}^{N,*}} \frac{p_t^N}{p_{t+1}^N} \right)^{1-\epsilon_N} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\left((\pi_t^N)^{\varrho_N} \pi_t^{1-\varrho_N} \right)^{\zeta_N} \bar{\pi}^{1-\zeta_N} \right]^{1-\epsilon_N}}{\pi_{t+1}^{1-\epsilon_N}} \frac{\pi_{t+1}^N}{\pi_{t+1}} \tilde{f}_{t+1}^N\end{aligned}\tag{D.53}$$

$$\begin{aligned}\tilde{f}_t^X &= \left(p_t^{X,*} \right)^{-\epsilon_X} m c_t^X y_t^X + \\ &\quad \beta a_t^{1-\sigma} \theta_X E_t \left(\frac{p_t^{X,*}}{p_{t+1}^{X,*}} \frac{p_t^X}{p_{t+1}^X} \right)^{-\epsilon_X} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\left((\pi_t^X)^{\varrho_X} \pi_t^{1-\varrho_X} \right)^{\zeta_X} \bar{\pi}^{1-\zeta_X} \right]^{-\epsilon_X}}{\pi_{t+1}^{-\epsilon_X}} \frac{\pi_{t+1}^X}{\pi_{t+1}} \tilde{f}_{t+1}^X\end{aligned}\tag{D.54}$$

$$\begin{aligned}\tilde{f}_t^M &= \left(p_t^{M,*} \right)^{-\epsilon_M} m c_t^M y_t^M + \\ &\quad \beta a_t^{1-\sigma} \theta_M E_t \left(\frac{p_t^{M,*}}{p_{t+1}^{M,*}} \frac{p_t^M}{p_{t+1}^M} \right)^{-\epsilon_M} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\left((\pi_t^M)^{\varrho_M} \pi_t^{1-\varrho_M} \right)^{\zeta_M} \bar{\pi}^{1-\zeta_M} \right]^{-\epsilon_M}}{\pi_{t+1}^{-\epsilon_M}} \frac{\pi_{t+1}^M}{\pi_{t+1}} \tilde{f}_{t+1}^M\end{aligned}\tag{D.55}$$

$$\begin{aligned}\tilde{f}_t^N &= \left(p_t^{N,*} \right)^{-\epsilon_N} m c_t^N y_t^N + \\ &\quad \beta a_t^{1-\sigma} \theta_N E_t \left(\frac{p_t^{N,*}}{p_{t+1}^{N,*}} \frac{p_t^N}{p_{t+1}^N} \right)^{-\epsilon_N} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\left((\pi_t^N)^{\varrho_N} \pi_t^{1-\varrho_N} \right)^{\zeta_N} \bar{\pi}^{1-\zeta_N} \right]^{-\epsilon_N}}{\pi_{t+1}^{-\epsilon_N}} \frac{\pi_{t+1}^N}{\pi_{t+1}} \tilde{f}_{t+1}^N\end{aligned}\tag{D.56}$$

$$\pi_t^X = \frac{p_t^X}{p_{t-1}^X} \pi_t\tag{D.57}$$

$$\pi_t^M = \frac{p_t^M}{p_{t-1}^M} \pi_t\tag{D.58}$$

$$\pi_t^N = \frac{p_t^N}{p_{t-1}^N} \pi_t\tag{D.59}$$

$$\pi_t^{SAE} = \frac{p_t^{SAE}}{p_{t-1}^{SAE}} \pi_t\tag{D.60}$$

$$1 = (1 - \theta_X) \left(p_t^{*,X} \right)^{1-\epsilon_X} + \theta_X \left[\left((\pi_{t-1}^X)^{\varrho_X} \pi_{t-1}^{1-\varrho_X} \right)^{\zeta_X} \bar{\pi}^{1-\zeta_X} \right]^{1-\epsilon_X} \left(\frac{1}{\pi_t^X} \right)^{1-\epsilon_X}\tag{D.61}$$

$$1 = (1 - \theta_M) \left(p_t^{*,M} \right)^{1-\epsilon_M} + \theta_M \left[\left((\pi_{t-1}^M)^{\varrho_M} \pi_{t-1}^{1-\varrho_M} \right)^{\zeta_M} \bar{\pi}^{1-\zeta_M} \right]^{1-\epsilon_M} \left(\frac{1}{\pi_t^M} \right)^{1-\epsilon_M} \quad (\text{D.62})$$

$$1 = (1 - \theta_N) \left(p_t^{*,N} \right)^{1-\epsilon_N} + \theta_N \left[\left((\pi_{t-1}^N)^{\varrho_N} \pi_{t-1}^{1-\varrho_N} \right)^{\zeta_N} \bar{\pi}^{1-\zeta_N} \right]^{1-\epsilon_N} \left(\frac{1}{\pi_t^N} \right)^{1-\epsilon_N} \quad (\text{D.63})$$

$$\left(\frac{R_t}{R} \right) = \left(\frac{R_{t-1}}{R} \right)^{\varrho_R} \left[\left(\frac{(\pi_t^{SAE})^{\alpha_\pi^{SAE}} \pi_t^{1-\alpha_\pi^{SAE}}}{\bar{\pi}} \right)^{\alpha_\pi} \left(\frac{gdp_t a_{t-1} / gdp_{t-1}}{a} \right)^{\alpha_Y} \right]^{1-\varrho_R} e_t^m \quad (\text{D.64})$$

$$c_t^{X,*} = \left(\frac{p_t^X}{rer_t} \right)^{-\epsilon^*} y_t^* \xi_t^{X,*} \quad (\text{D.65})$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^S \pi_t^*}{\pi_t} \quad (\text{D.66})$$

$$p_{m,t} = rer_t p_{m,t}^* \quad (\text{D.67})$$

$$R_t^* = R_t^W \exp \left\{ \phi_B \left(\bar{b} - \frac{b_t^* rer_t}{p_t^Y gdp_t} \right) \right\} \xi_t^{R1} \xi_t^{R2} \quad (\text{D.68})$$

$$i_t = i_t^X + i_t^N \quad (\text{D.69})$$

$$h_t^X = \Delta_t^{WX} h_t^{X,d} \quad (\text{D.70})$$

$$h_t^N = \Delta_t^{WN} h_t^{N,d} \quad (\text{D.71})$$

$$y_t^N = \Delta_t^N (c_t^N + g_t + \tilde{i}_t^N) \quad (\text{D.72})$$

$$y_t^X = \Delta_t^X (c_t^X + \tilde{i}_t^X + c_t^{X,*}) \quad (\text{D.73})$$

$$y_t^M = \Delta_t^M (c_t^M + \tilde{i}_t^M + m_t^X + m_t^N) \quad (\text{D.74})$$

$$\Delta_t^{WX} = (1 - \theta_{WX}) \left(w_t^{X,*} \right)^{-\epsilon_W} + \theta_{WX} \left(\frac{w_{t-1}^X}{w_t^X} \frac{a}{a_{t-1}} \frac{((\pi_{t-1}^X)^{\varrho_{WX}} \pi_{t-1}^{1-\varrho_{WX}})^{\varsigma_{WX}} \bar{\pi}^{1-\varsigma_{WX}}}{\pi_t} \right)^{-\epsilon_W} \Delta_{t-1}^{WX} \quad (\text{D.75})$$

$$\Delta_t^{WN} = (1 - \theta_{WN}) \left(w_t^{N,*} \right)^{-\epsilon_W} + \theta_{WN} \left(\frac{w_{t-1}^N}{w_t^N} \frac{a}{a_{t-1}} \frac{((\pi_{t-1}^N)^{\varrho_{WN}} \pi_{t-1}^{1-\varrho_{WN}})^{\varsigma_{WN}} \bar{\pi}^{1-\varsigma_{WN}}}{\pi_t} \right)^{-\epsilon_W} \Delta_{t-1}^{WN} \quad (\text{D.76})$$

$$\Delta_t^X = (1 - \theta_X) \left(p_t^{*,X} \right)^{-\epsilon_X} + \theta_X \left(\frac{((\pi_{t-1}^X)^{\varrho_X} \pi_{t-1}^{1-\varrho_X})^{\varsigma_X} \bar{\pi}^{1-\varsigma_X}}{\pi_t^X} \right)^{-\epsilon_X} \Delta_{t-1}^X \quad (\text{D.77})$$

$$\Delta_t^M = (1 - \theta_M) \left(p_t^{*,M} \right)^{-\epsilon_M} + \theta_M \left(\frac{((\pi_{t-1}^M)^{\varrho_M} \pi_{t-1}^{1-\varrho_M})^{\varsigma_M} \bar{\pi}^{1-\varsigma_M}}{\pi_t^M} \right)^{-\epsilon_M} \Delta_{t-1}^M \quad (\text{D.78})$$

$$\Delta_t^N = (1 - \theta_N) \left(p_t^{*,N} \right)^{-\epsilon_N} + \theta_N \left(\frac{\left((\pi_{t-1}^N)^{\varrho_N} \pi_{t-1}^{1-\varrho_N} \right)^{\varsigma_N} \bar{\pi}^{1-\varsigma_N}}{\pi_t^N} \right)^{-\epsilon_N} \Delta_{t-1}^N \quad (\text{D.79})$$

$$tb_t = rer_t p_t^{Co,*} y_t^{Co} \frac{a_{t-1}^{Co}}{a_{t-1}} + p_t^X c_t^{X,*} - p_{m,t} m_t \quad (\text{D.80})$$

$$a_t^{Co} = \left(\frac{a_{t-1}^{Co}}{a_{t-1}} \right)^{1-\Gamma_{Co}} a_t^{\Gamma_{Co}} \quad (\text{D.81})$$

$$rer_t b_t^* = tb_t + \frac{rer_t}{\pi_t^* a_{t-1}} R_{t-1}^* b_{t-1}^* - (1 - \vartheta) rer_t p_t^{Co,*} y_t^{Co} \frac{a_{t-1}^{Co}}{a_{t-1}} \quad (\text{D.82})$$

$$gdp_t = c_t + g_t + i_t + c_t^{X,*} + y_t^{Co} \frac{a_{t-1}^{Co}}{a_{t-1}} - m_t \quad (\text{D.83})$$

$$p_t^Y gdp_t = c_t + p_t^G g_t + p_t^I i_t + tb_t \quad (\text{D.84})$$

D.4.1 Steady State

The given endogenous are: $\{R, h^X, h^N, p^X/p^I, p^M/p^I, s^{Co} = rer p^{Co,*} y^{Co}/(p^Y gdp), s^M = p_m y^M/(p^Y gdp), s^g = p^N g/(p^Y gdp)\}^{52}$ and the exogenous variables or parameters that are calculated endogenously are: $\{\beta, \tilde{\xi}^{h,N}, z^X, g, y^*, \pi^*, y^{Co}, \gamma, \bar{b}\}$. The rest of the steady state values of the exogenous variables (not endogenously determined nor listed in table D.2) are normalized to one.

By (D.68) (assuming that the part inside the bracket is zero):

$$R^* = R^W \xi^{R1}$$

By (D.49)

$$a^X = a^{\frac{2\Gamma_X - 1}{\Gamma_X}}$$

By (D.81)

$$a^{Co} = a^{\frac{2\Gamma_{Co} - 1}{\Gamma_{Co}}}$$

By (D.64) and (D.60) (assuming $\epsilon^m = 1$):

$$\pi^{SAE} = \pi = \bar{\pi}$$

By (D.8):

$$\beta = \frac{a^\sigma \pi}{R}$$

By (D.9):

$$\pi^S = \frac{a^\sigma \pi}{R^* \beta}$$

By (D.66):

$$\pi^* = \frac{\pi}{\pi^S}$$

By (D.67)-(D.69):

$$\pi^X = \pi^M = \pi^N = \pi$$

⁵²The values for h^X, h^N, s^M were set to get as close as possible to the targets for $\gamma, tby, p^N y^N / p^Y gdp$.

By (D.61)-(D.63):

$$p^{X,*} = p^{M,*} = p^{N,*} = 1$$

By (D.20)-(D.21):

$$w^{X,*} = w^{N,*} = 1$$

By (D.75)-(D.79):

$$\Delta^{WX} = \Delta^{WN} = \Delta^X = \Delta^M = \Delta^N = 1$$

By (D.51)-(D.56)

$$mc^X = \frac{\epsilon_X - 1}{\epsilon_X}$$

$$mc^M = \frac{\epsilon_M - 1}{\epsilon_M}$$

$$mc^N = \frac{\epsilon_N - 1}{\epsilon_N}$$

By (D.16)-(D.19)

$$\mu^{WX} = \mu^{WN} = \frac{\epsilon_W - 1}{\epsilon_W}$$

By (D.70)-(D.71):

$$h^{X,d} = h^X$$

$$h^{N,d} = h^N$$

From the relative prices p^X/p^I and p^M/p^I , we get using (D.28)-(D.30) the relative prices:

$$\begin{aligned} \frac{p^{TI}}{p^I} &= \left(\frac{p^X}{p^I} \right)^{\gamma_{TI}} \left(\frac{p^M}{p^I} \right)^{(1-\gamma_{TI})} \\ \frac{p^N}{p^I} &= \left(\frac{1 - (1 - \gamma_I) (p^{TI}/p^I)^{1-\varrho_I}}{\gamma_I} \right)^{\frac{1}{1-\varrho_I}} \\ \frac{p^T}{p^I} &= \left(\frac{p^X}{p^I} \right)^{\gamma_T} \left(\frac{p^M}{p^I} \right)^{(1-\gamma_T)} \end{aligned}$$

From (D.12)-(D.13):

$$\frac{\tilde{\mu}^X}{p^I} = \frac{\tilde{\mu}^N}{p^I} = 1/u$$

By (D.10)-(D.11):

$$R^X = \frac{(\tilde{\mu}^X/p^I)(1 - \beta a^{-\sigma}(1 - \delta))}{\beta a^{-\sigma}(p^X/p^I)}$$

$$R^N = \frac{(\tilde{\mu}^N/p^I)(1 - \beta a^{-\sigma}(1 - \delta))}{\beta a^{-\sigma}(p^N/p^I)}$$

By (D.35):

$$\frac{p_m}{p^I} = mc^M(p^M/p^I)$$

By (D.67):

$$\frac{rer}{p^I} = \frac{p_m/p^I}{p_m^*}$$

It is further assumed that $p^A = p^E = p^T$, and so, we also have p^A/p^I and p^E/p^I . By (D.50):

$$\frac{p^{ME}}{p^I} = \left(\frac{p^M}{p^I}\right)^{1-\gamma_{EF}} \left(\frac{p^T}{p^I}\right)^{\gamma_{EF}}$$

By (D.47)-(D.48):

$$mc^{V,X} = \left(\frac{mc^X (p^X/p^I)^{1-\gamma_X} (1-\gamma_X)^{1-\gamma_X} \gamma_X^{\gamma_X}}{(p^{ME}/p^I)^{1-\gamma_X}}\right)^{\frac{1}{\gamma_X}}$$

$$mc^{V,N} = \left(\frac{mc^N (p^N/p^I)^{1-\gamma_N} (1-\gamma_N)^{1-\gamma_N} \gamma_N^{\gamma_N}}{(p^{ME}/p^I)^{1-\gamma_N}}\right)^{\frac{1}{\gamma_N}}$$

By (D.42):

$$\frac{w^N}{p^I} = \left(\frac{mc^{V,N} z^N a^{1-\alpha_N} (p^N/p^I) (1-\alpha_N)^{1-\alpha_N} \alpha_N^{\alpha_N}}{((p^N/p^I) R^N)^{\alpha_N}}\right)^{\frac{1}{1-\alpha_N}}$$

By (D.7):

$$\frac{\tilde{\xi}^{h,N}}{p^I} = \frac{\mu^{WN}}{(h^N)^\varphi} \frac{w^N}{p^I}$$

Assuming that $\tilde{\xi}^{h,X} = \tilde{\xi}^{h,N}$, we also have $\tilde{\xi}^{h,X}/p^I$ and with (D.6):

$$\frac{w^X}{p^I} = \frac{(\tilde{\xi}^{h,X}/p^I)(h^X)^\varphi}{\mu^{WX}}$$

By (D.41):

$$z^X = \frac{((p^X/p^I) R^X)^{\alpha_X} (w^X/p^I)^{1-\alpha_X}}{mc^{V,X} (a^X)^{1-\alpha_X} (p^X/p^I) (1-\alpha_X)^{1-\alpha_X} \alpha_X^{\alpha_X}}$$

By (D.37) and (D.39):

$$v^X = h^{X,d} z^X (a^X)^{1-\alpha_X} \left[\frac{\alpha_X}{1-\alpha_X} \frac{w^X/p^I}{(p^X/p^I) R^X} \right]^{\alpha_X}$$

$$v^N = h^{N,d} z^N a^{1-\alpha_N} \left[\frac{\alpha_N}{1-\alpha_N} \frac{w^N/p^I}{(p^N/p^I) R^N} \right]^{\alpha_N}$$

By (D.38) and (D.40):

$$k^X = a \frac{v^X}{z^X (a^X)^{1-\alpha_X}} \left[\frac{\alpha_X}{1-\alpha_X} \frac{w^X/p^I}{(p^X/p^I) R^X} \right]^{1-\alpha_X}$$

$$k^N = a \frac{v^N}{z^N a^{1-\alpha_N}} \left[\frac{\alpha_N}{1-\alpha_N} \frac{w^N/p^I}{(p^N/p^I) R^N} \right]^{1-\alpha_N}$$

By (D.43) and (D.45):

$$y^X = v^X \left[\frac{\gamma_X}{1-\gamma_X} \frac{p^{ME}/p^I}{mc^{V,X}} \frac{1}{p^X/p^I} \right]^{-(1-\gamma_X)}$$

$$y^N = v^N \left[\frac{\gamma_N}{1-\gamma_N} \frac{p^{ME}/p^I}{mc^{V,N}} \frac{1}{p^N/p^I} \right]^{-(1-\gamma_N)}$$

By (D.44) and (D.46):

$$m^X = y^X \left[\frac{1 - \gamma_X}{\gamma_X} \frac{mc^{V,X}}{p^{ME}/p^I} p^X / p^I \right]^{\gamma_X}$$

$$m^N = y^N \left[\frac{1 - \gamma_N}{\gamma_N} \frac{mc^{V,N}}{p^{ME}/p^I} p^N / p^I \right]^{\gamma_N}$$

By (D.51) and (D.53):

$$\tilde{f}^X = \frac{\epsilon_X - 1}{\epsilon_X} \frac{y^X}{(1 - \beta a^{1-\sigma} \theta_X)}$$

$$\tilde{f}^N = \frac{\epsilon_N - 1}{\epsilon_N} \frac{y^N}{(1 - \beta a^{1-\sigma} \theta_N)}$$

By (D.14)-(D.15):

$$i^X = \frac{k^X}{u} \left(1 - \frac{1 - \delta}{a} \right)$$

$$i^N = \frac{k^N}{u} \left(1 - \frac{1 - \delta}{a} \right)$$

By (D.69):

$$i = i^X + i^N$$

By (D.31)-(D.34):

$$\tilde{i}^N = \gamma_I \left(\frac{p^N}{p^I} \right)^{-\varrho_I} i$$

$$\tilde{i}^T = (1 - \gamma_I) \left(\frac{p^{TI}}{p^I} \right)^{-\varrho_I} i$$

$$\tilde{i}^X = \gamma_{TI} \left(\frac{p^{TI}/p^I}{p^X/p^I} \right) \tilde{i}^T$$

$$\tilde{i}^M = (1 - \gamma_{TI}) \left(\frac{p^{TI}/p^I}{p^M/p^I} \right) \tilde{i}^T$$

When replacing equations (D.72)-(D.74) into equation (D.84) (and using the identities of expenditures), one gets an alternative sum for nominal gdp :

$$p^Y gdp = p^X y^X + rer p^{Co,*} y^{Co} \frac{a^{Co}}{a} + p^N y^N + p^M y^M - p^M (m^X + m^N) - p_m m$$

which can also be written in terms of prices relative to investment:

$$\frac{p^Y}{p^I} gdp = \frac{p^X}{p^I} y^X + \frac{rer}{p^I} p^{Co,*} y^{Co} \frac{a^{Co}}{a} + \frac{p^N}{p^I} y^N + \frac{p^M}{p^I} y^M - \frac{p^M}{p^I} (m^X + m^N) - \frac{p_m}{p^I} m$$

And using s^{Co}, s^M :

$$\frac{p^Y}{p^I} gdp = \frac{\frac{p^X}{p^I} y^X + \frac{p^N}{p^I} y^N - \frac{p^M}{p^I} (m^X + m^N)}{1 - s^{Co} - s^M \frac{((p^M - p_m)/p^I)}{p_m/p^I}}$$

With this, we can get:

$$y^{Co} = \frac{s^{Co} (p^Y/p^I) gdp}{(rer/p^I) p^{Co,*} \frac{a}{a^{Co}}}$$

$$y^M = \frac{s^M(p^Y/p^I)gdp}{p_m/p^I}$$

$$g = \frac{s^g(p^Y/p^I)gdp}{p^N/p^I}$$

By (D.55):

$$\tilde{f}^M = \frac{\epsilon_M - 1}{\epsilon_M} \frac{y^M}{(1 - \beta a^{1-\sigma} \theta_M)}$$

By (D.36):

$$m = y^M$$

By (D.72):

$$c^N = y^N - g - \tilde{i}^N$$

By (D.74):

$$c^M = y^M - \tilde{i}^M - m^X - m^N$$

By (D.25):

$$c^T = \frac{c^M}{1 - \gamma_T} \left(\frac{p^M/p^I}{p^T/p^I} \right)$$

By (D.24):

$$c^X = \gamma_T \left(\frac{p^T/p^I}{p^X/p^I} \right) c^T$$

By (D.22)-(D.23):

$$\gamma = \frac{(p^N/p^I)^{\varrho} c^N}{(p^T/p^I)^{\varrho} c^T + (p^N/p^I)^{\varrho} c^N}$$

By (D.26)-(D.27):

$$\frac{p^{SAE}}{p^I} = \left[(1 - \gamma) \left(\frac{p^T}{p^I} \right)^{1-\varrho} + \gamma \left(\frac{p^N}{p^I} \right)^{1-\varrho} \right]^{\frac{1}{1-\varrho}}$$

$$p^I = \left[\left(\frac{p^{SAE}}{p^I} \right)^{1-\gamma_{AC}-\gamma_{EC}} \left(\frac{p^T}{p^I} \right)^{\gamma_{AC}+\gamma_{EC}} \right]^{-1}$$

Now, we get all prices by multiplying the price relative to investment by p^I :
 $\{p^X, p^M, p^N, p^T, p^{TI}, p^{SAE}, p^{ME}, rer, w^X, w^N, \tilde{\mu}^X, \tilde{\mu}^N, p_m, \tilde{\xi}^{h,N}\}$

By (D.22):

$$c = \frac{1}{\gamma} (p^N)^{\varrho} c^N$$

(also check equation $c = c^T (p^T)^{\varrho} / (1 - \gamma)$)

By (D.73):

$$c^{X,*} = y^X - \tilde{i}^X - c^X$$

By (D.65):

$$y^* = \frac{c^{X,*}}{\xi^{X,*}} \left(\frac{p^X}{rer} \right)^{\epsilon^*}$$

By (D.83):

$$gdp = c + g + i + c^{X,*} + y^{Co} \frac{a^{Co}}{a} - m$$

$$p^Y = \frac{p^Y gdp}{gdp}$$

By (D.80):

$$tb = rer\ p^{Co,*} y^{Co} \frac{a^{Co}}{a} + p^X c^{X,*} - p_m m$$

By (D.82):

$$b^* = \frac{tb - (1 - \vartheta) rer\ p^{Co,*} y^{Co} \frac{a^{Co}}{a}}{rer\left(1 - \frac{R^*}{\pi^* a}\right)}$$

By (D.68) (part that was assumed zero):

$$\bar{b} = \frac{b^* rer}{p^Y gdp}$$

By (D.5):

$$\tilde{\lambda} = \xi^\beta c^{-\sigma} \left(1 - \frac{\phi_C}{a}\right)^{-\sigma}$$

By (D.18)-(D.19):

$$\begin{aligned} \tilde{f}^{WX} &= \frac{\mu^{WX} \tilde{\lambda} h^{X,d}}{1 - \theta_{WX} a^{1-\sigma} \beta} \\ \tilde{f}^{WN} &= \frac{\mu^{WN} \tilde{\lambda} h^{N,d}}{1 - \theta_{WN} a^{1-\sigma} \beta} \end{aligned}$$

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